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INTERNATIONAL WORKSHOP ON OLIGOPOLY THEORY

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Host Department of Economics, Chonnam National University, Korea

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Education of Researchers for an Inclusive Regional Innovation and the Sustainable Future

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Hosted by Department of Economics, Chonnam National University, Korea
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Program

14:00-14:05 Opening Remark
Prof. Chanyoung Lee (Department Head, Chonnam National University)

14:05-14:55 Keynote Speech
Chair: Chul-Hi Park (Chonnam National University, Korea)
Speaker: Prof. Toshihiro Matsumura (The University of Tokyo, Japan)
Title: Tax versus Regulations: An Analysis of Robustness to Polluter Lobbying against Near-Zero Emission Targets

15:00-17:40 Session Presentation
Chair: Prof. Sang-Ho Lee (Chonnam National University, Korea)

15:00-15:40
Speaker: Yuta Yasui (University of California, Los Angeles, U.S.A.)
Title: Controlling Fake Reviews

15:40-16:20
Speaker: Prof. Seung-Leul Kim (Kangwon National University, Korea)
Title: Optimal Tariff Policies with Emission Taxes under Non-Restrictive Two-part Licensing Strategies by a Foreign Eco-Competitor

16:20-17:00
Speaker: Prof. Susumu Sato (Hitotsubashi University, Japan)
Title: Usage Lock-In and Platform Competition under Multihoming

17:00-17:40
Speaker: Jiaqi Chen (Chonnam National University, Korea)
Title: Comparisons of Output Subsidy and R&D Subsidy in a Differentiated Market

17:40-17:50 Closing Remark
Prof. Sang-Ho Lee (Chonnam National University, Korea)

Tax versus Regulations: An Analysis of Robustness to Polluter Lobbying against Near-Zero Emission Targets*

Kosuke Hirose†, Akifumi Ishihara‡, and Toshihiro Matsumura§

April 19, 2021

Abstract

We investigate polluter lobbying against near-zero emission targets in monopoly markets. We compare three typical environmental policies, an emission cap regulation that restricts total emissions, an emission intensity regulation that restricts emissions per unit of output, and an emission tax. We presume that a policy is most robust to lobbying when a lesser strict emission target (an increase of the targeted emission level) the government imposes to the industry increases the firm's profit least significantly under the policy among the three policies. We find that the emission tax is the most robust among the three policies in the presence of lobbying if the government aims for a net-zero emission society.

JEL classification codes: Q52, L13, L51

Keywords: net-zero emission industry, emission cap, emission intensity, emission tax, emission equivalence, profit ranking

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1 Introduction

Global warming is one of the most serious risks that society faces. Recently, many countries have voluntarily committed to reducing CO2 emissions under the Paris Agreement on climate change. Moreover, several European countries have declared that they aim to achieve net-zero emission societies, and China and Japan have followed suit.[1] To achieve this goal, several industries that emit huge amounts of CO2, such as electric power, steel, and transportation, may face near-zero emission constraints imposed by authorities. For example, the US president Joe Biden signed a new executive order with a commitment to build a carbon pollution-free electricity sector by 2035, and reach net zero emissions by 2050 in whole sectors.[2] However, such strict policies substantially reduce the profit of firms, and thus, firms may have strong incentives to lobby against restrictions and in favor of weaker regulation that increases the upper limit of the industry's emissions. Environmental policies affect an industry's profits, and firms often try to influence policymakers' behavior (Lowry, 1992; Engel, 1997), and ambitious environmental policy may not be implementable.

In this study, we presume that firms have stronger incentives of lobbying to manipulate emission targets when a lesser strict emission policy (an increase of the upper limit of emission) increases the firms' profit more significantly, and investigate under which policy among typical environmental policies the firms' lobbying incentives are weakest (and thus the policy is the most robust to the polluter lobbying). For this purpose, we consider a monopoly industry and investigate the relationship between targeted emission level and the monopoly profit.[3] We compare three environmental policies that are intensively discussed in the literature: emission cap regulation, emission intensity regulation, and emission tax.

[1] Reuters, https://jp.reuters.com/article/japan-politics-suga/japan-aims-for-zero-emissions-carbon-neutral-society-by-2050-pm-idUSKBN27B0FB

[2] Energy live news, https://www.energylivenews.com/2021/01/28/biden-wants-carbon-free-electricity-by-2035/

[3] We can derive similar policy implications in symmetric Cournot oligopolies, and in a symmetric Bertrand oligopoly in a differentiated product market, under the standard assumptions in this field.

We find that when the targeted emission level is close to zero, an increase in the targeted emission level most (least) significantly raises the monopoly profit under the emission cap (emission tax) regulation. Then, in cases of near-zero emission target, the emission tax is the most robust to the polluter lobbying because the firm has the weakest incentive for manipulating targets. By contrast, when the targeted emission level is very far from zero-emission, the emission tax policy is the most vulnerable policy to the polluter lobbying.

Our result is consistent with the environmental policies adopted in Japan. Until recently, the emission target was much less strict in Japan than in European countries. The Japanese government have mainly used emission cap and emission intensity regulations as environmental policy tools and did not introduce the effective emission tax in the presence of aggressive lobbying by major industry groups. Recently, the new Japanese Cabinet, the Suga Cabinet, declared a net-zero emissions goal by 2050 for Japan, and at the same time has initiated intensive discussion about the introduction of carbon pricing.[4] These policy choices are supported by our result in terms of robustness to industry lobbying.

The three environmental policies investigated in this paper are intensively discussed in the literature (Amir et al.,2018; Alesina and Passarelli, 2014; Barnett, 1980; Baumol and Oates, 1988; Besanko, 1987; Helfand,1991; Holland, 2012; Katsoulacos and Xepapadeas, 1996; Lahiri and Ono, 2007; Lee, 1999). Several studies have discussed welfare ranking among these environmental policy measures. In perfectly competitive markets, Pigovian taxes yield the first best (Pigou, 1932), while emission intensity regulations do not (Holland, 2012; Holland et al., 2009). This implies that emission taxes are the best for welfare. However, in imperfectly competitive markets, the first best is not implementable by emission taxes owing to underproduction (Buchanan, 1969; Katsoulacos and Xepapadeas, 1996) and emission regulations may be better for welfare than an emission tax (Amir et al., 2018;

[4]Reuters, https://www.reuters.com/article/japan-economy-climate-change/japan-advisers-urge-quick-adoption-of-carbon-pricing-to-hit-emissions-goal-idINL4N2KU3H6

Helfand, 1991; Holland, 2009; Kiyono and Ishikawa, 2013; Li and Shi 2015; Montero, 2002). Lahiri and Ono (2007) consider the case in which emission targets are close to business-as-usual levels and show that an emission intensity regulation may be better than an emission tax. Hirose and Matsumura (2020) show that when emission targets are close to zero, the emission intensity regulation dominates the emission cap regulation and the emission tax, whereas the inverse may hold when emission targets are moderate. However, no study investigates the threat of polluter lobbying.

Aidt (1998, 2010) and Cai and Li (2020) adopt the approach of Grossman and Helpman (1994) to investigate polluters' lobbying activity in imperfectly competitive markets. They show the relationship between lobbying intensity and firms' characteristics. However, they do not compare lobbying activities under the typical environmental policy measures discussed in this study. Moreover, to the best of our knowledge, no work has presented a clear policy ranking against polluter lobbying when a government aims to achieve near-zero emissions.

2 The Model

We consider an industry with a polluting monopolist. The firm produces a single commodity for which the inverse demand function is given by $P : \mathbb{R}_+ \mapsto \mathbb{R}_+$. Let $c(q, x) : \mathbb{R}_+^2 \mapsto \mathbb{R}_+^2$ be the cost function, where q is the output and x is the abatement level. Let $e(q, x) : \mathbb{R}_+^2 \mapsto \mathbb{R}_+^2$ be the pollution emission level. We assume that P, c, and e are twice continuously differentiable and satisfy $P' < 0$ as long as $P > 0$, $P' + P''q < 0$, $c_q > 0$, $c_x > 0$, $c_{qq} \geq 0$, $c_{xx} > 0$, $e_q > 0$, $e_x < 0$, $e_{qq} \geq 0$, and $e_{xx} > 0$ for $q, x > 0$, where subscripts denote derivatives (e.g., $c_q = \partial c/\partial q$ and $c_{qq} = \partial^2 c/\partial q^2$). We also assume that $P(0) - c_q(0, x)$ is sufficiently large, $c_x(q, 0)$ is sufficiently small, and $|c_{qx}|$ and $|e_{qx}|$ are sufficiently small relative to c_{qq}, c_{xx}, e_{qq}, or e_{xx} which ensure that the solution characterized below is interior and satisfies the second-order conditions. These are standard assumptions in the literature (Carraro et al., 1996).

We consider three environmental policies that aim to restrict the total emission not greater than the emission target E. The first is an emission cap regulation in which the monopolist chooses q and x under the constraint $e \leq E$. The second policy is an emission intensity regulation in which the monopolist chooses q and x under the constraint $e/q \leq \alpha$ and the government chooses α such that the equilibrium emission is equal to E. The last policy is an emission tax in which the government chooses the emission tax rate t such that the equilibrium emission is equal to E. The firm's profit is $P(q)q - c(q,x)$ when the emission cap or emission intensity regulation is imposed, and it is $P(q)q - c(q,x) - te$ when the emission tax is adopted.

Let $\pi^C(E)$, $\pi^I(E)$, and $\pi^T(E)$ be the firm's optimal profit when the emission target is E under the emission cap regulation, the emission intensity regulation, and the emission tax, respectively. If the emission target is initially $E = E^o$ and is relaxed to $E^r(> E^o)$, then under policy $i(= C, I, T)$, the firm increases the profit by $\pi^i(E^r) - \pi^i(E^o)$. This implies that if the firm can manipulate the emission target from E^o to E^r through lobbying, then the firm is willing to pay $\pi^i(E^r) - \pi^i(E^o)$ for the lobbying.[5] Hence, we presume the incremental profit to be the firm's lobbying incentive and say that the policy is more robust to lobbying as the incremental profit is smaller.

Let E^B be the emission when the firm maximizes its profit without either type of emission regulation or the emission tax (the superscript B denotes business as usual). If $E \geq E^B$, the constraint is not effective (non-binding). Throughout the analysis, we assume $E \in [0, E^B)$.

[5] In lobbying models such as Grossman and Helpman (1994), to pay the incremental increase in the payoff is demonstrated as an equilibrium behavior.

3 Analysis of Three Environmental Policies

3.1 Emission cap regulation

First, we consider the emission cap regulation. The government imposes the upper bound of the total emission, $E \in [0, E^B)$. The firm chooses q and x to maximize its profit under the constraint $e(q, x) \leq E$. The firm's optimization problem is

$$\begin{aligned} &\max_{q,x} P(q)q - c(q,x) \\ &s.t. \;\; e(q,x) \leq E. \end{aligned} \tag{1}$$

Because we assume $E < E^B$, the constraint must be binding (i.e., $e(q, x) = E$ in equilibrium). Then, once the firm chooses q, x is automatically determined by the constraint $e(q, x) = E$. Let $\hat{x}(q, E)$ be the value that satisfies $e(q, \hat{x}(q, E)) \equiv E$. As the firm mechanically chooses $x = \hat{x}(q, E)$ given q, substituting this constraint into the profit function yields

$$P(q)q - c(q, \hat{x}(q, E)). \tag{2}$$

Note that $\partial \hat{x}/\partial q = -e_q/e_x$ due to the implicit function theorem. Then, the optimal choice, denoted by (q^C, x^C), is characterized by the following first-order condition

$$P + P'q - c_q + c_x \frac{e_q}{e_x} = 0 \tag{3}$$

and $e(q^C, x^C) = E$. In the first-order condition, the marginal production cost is $c_q + c_x(-e_q/e_x)$. A marginal increase in q increases e by e_q. To cancel this increase in emissions, the firm must increase x by $(-e_q/e_x)$, which increases the cost by $c_x(-e_q/e_x)$.

3.2 Emission intensity regulation

Next, we consider the emission intensity regulation. The government imposes the upper bound of emission per unit of output, α. The firm chooses q and x to maximize its profit under the constraint $e(q,x) \leq \alpha q$. The firm's optimization problem is

$$\begin{aligned} &\max_{q,x} P(q)q - c(q,x) \\ &s.t. \quad e(q,x) \leq \alpha q. \end{aligned} \tag{4}$$

When the constraint is binding, similar to the emission cap regulation, the abatement level is determined as $x = \hat{x}(q, \alpha q)$ given α and q. Substituting this constraint into the profit function yields $p(q)q - c(q, \hat{x}(q, \alpha q))$. By taking the derivative with respect to q, the firm's optimal choice, denoted by (q^I, x^I) satisfies the following single first-order condition

$$P'q + P - c_q - c_x \frac{\alpha - e_q}{e_x} = 0 \tag{5}$$

and the constraint $e(q^I, x^I) = \alpha q^I$. In addition, if the government chooses α to induce the total emission to be $E \in (0, E^B)$, the intensity, denoted by α^I, satisfies $e(q^I, x^I) = \alpha^I q^I = E$ as well.

The difference from the emission cap regulation is characterized as follows.

Lemma 1. *(i)* $\alpha^I = 0$ *and* $(q^C, x^C) = (q^I, x^I)$ *for* $E = 0$. *(ii)* $\alpha^I < E/q^C$ *for* $E \in (0, E^B)$.

Proof. See Appendix. □

When $E = 0$, the per-output emission level is $\alpha^I = 0$ under the emission intensity regulation. Then, the total emission level becomes $\alpha^I q = 0$, which is independent of the output and the same as the emission cap regulation. Therefore, the firm faces the same constraint under the emission cap regulation and the emission intensity regulation given $E = 0$.

However, as long as $E > 0$, under the emission intensity regulation, $\alpha^I > 0$ and firm chooses q and x given α, not e. The total emission $\alpha^I q$ is increasing in q, in contrast to the case of emission cap regulation. Thus, the firm has a stronger incentive to increase q under the emission intensity regulation than under the emission cap regulation (Holland et al., 2009; Ino and Matsumura, 2019). Therefore, if the government sets $\alpha = E/q^C$, the resulting emission exceeds E. Given the firm's choice being expected, the government chooses a lower emission intensity (i.e., $\alpha^I < E/q^C$) to realize emission E (Hirose and Matsumura, 2020).

3.3 Emission tax

Finally, we consider the emission tax. Given that the government imposes emission tax t, the firm chooses q and x to maximize its after-tax profit. The firm's optimization problem is

$$\max_{q,x} P(Q)q - c(q,x) - te(q,x). \tag{6}$$

The firm's optimal choice, denoted by (q^T, x^T), satisfies the following first-order conditions:

$$\frac{\partial \pi}{\partial q} = P'q + P - c_q - te_q = 0, \tag{7}$$

$$\frac{\partial \pi}{\partial x} = -c_x - te_x = 0. \tag{8}$$

In addition, if the government attempts to induce the total emission equal to E, then the emission tax t is determined to satisfy $e(q^T, x^T) = E$. Substituting (8) ($t = -c_x/e_x$) into (8), we find that (7) is expressed as (3). This implies that the firm chooses the same output and abatement levels as those under the emission cap regulation. This result is a straightforward application of the well-known tariff-quota equivalence.

Lemma 2. *$q^T = q^C$ and $x^T = x^C$ for all E.*

3.4 Results

We now investigate the firm's lobbying incentive when the government aims $E = 0$ to realize a zero-emission society. Under policy $i(= C, I, T)$, if the firm lobbies to manipulate the target to E^r, then the firm can increase the profit by $\pi^i(E^r) - \pi^i(0)$, which is the firm's lobbying incentive. The lobbying incentives can be ranked by the following proposition.

Proposition 1. *(i)* $\pi^C(0) = \pi^I(0) = \pi^T(0)$, *(ii)* $\pi^C(E) > \pi^I(E)$ *and* $\pi^C(E) > \pi^T(E)$ *for* $E \in (0, E^B)$. *(iii) There exists* $\hat{E}_0 > 0$ *such that* $\pi^I(E) > \pi^T(E)$ *for all* $E \in (0, \hat{E}_0)$.

Proof. See Appendix. □

Proposition 1(i,ii) states that an increase in E from $E = 0$ mostly increases the firm's profit under the emission cap regulation.

The comparison between $\pi^C(E)$ and $\pi^I(E)$ is implied by Lemma 1. When $E = 0$,both regulations yield the same outcome. When $E > 0$, under the emission intensity regulation, the firm has a stronger incentive to expand its output, and expecting this ex post aggressive behavior of the firm, the government sets a stricter regulation (i.e., $\alpha^I < E/q^C$), which leads to $\pi^C(E) > \pi^I(E)$ for $E \in (0, E^B)$.

Recall from Lemma 2 that the emission cap regulation and the emission tax yield the same outcomes. Thus, the difference in profit between the two policies is te^T, which is zero when $E = 0$ and positive when $E = e^T > 0$. These lead to $\pi^C(0) = \pi^T(0)$ and $\pi^C(E) > \pi^T(E)$ for $E \in (0, E^B)$.

To understand Proposition 1(iii), we can use the property on the derivative $d\pi^i/dE$. The lobbying incentive can be expressed as $\pi^i(E^r) - \pi^i(E^o) = \int_{E^o}^{E^r} (d\pi^i/dE)dE$. By the envelope theorem,

$$\begin{aligned} \frac{d\pi^C}{dE} &= -\frac{c_x(q^C, \hat{x}(q^C, E))}{e_x(q^C, \hat{x}(q^C, E))}, \\ \frac{d\pi^I}{dE} &= -\frac{c_x(q^I, \hat{x}(q^I, \alpha^I q^I))}{e_x(q^I, \hat{x}(q^I, \alpha^I q^I))}, \\ \frac{d\pi^T}{dE} &= -e(q^T, x^T)\frac{dt}{dE}. \end{aligned} \tag{9}$$

Under the emission cap regulation and the emission intensity regulation, a marginal increase in E improves the profit through a marginal reduction in the abatement level. When $E = 0$, as the abatement level is positive, the marginal cost of abatement is also positive, which implies that a marginal increase of the profit is positive at $E = 0$. By contrast, under the emission tax, a marginal increase in E improves the profit through a marginal reduction in the tax rate. Nevertheless, when $E = 0$, since the firm implements zero emission (and thus the tax payment is zero), a marginal increase of the profit becomes zero. Accordingly, we obtain the following supplementary result that directly leads to Proposition 1(iii).

Lemma 3.

$$\frac{d\pi^C}{dE}\bigg|_{E=0} = \frac{d\pi^I}{dE}\bigg|_{E=0} > \frac{d\pi^T}{dE}\bigg|_{E=0} = 0.$$

Proof. See Appendix. □

Henceforth, we discuss the difference in profit when E is zero and when E is (small) positive. However, a marginal increase of the profits ($d\pi/dE$) itself may be important for discussing lobbying incentives when the firm can choose lobbying level (and the emission target level) continuously. Suppose that the government initially wants to implement emission target E^o, the realized emission target after lobbying is E^r, and lobbying cost is $L(\Delta E)$ ($\Delta E \equiv E^r - E^o$). Suppose that $L' > 0$ and L'' is sufficiently large that the second-order condition is satisfied. Then, the realized E^r is determined by $d\pi^i/dE = L'(\Delta E)$, $E = E^r$ and $\Delta E = E^r - E^o$ for $i = C, I, T$. If other conditions are equal, the larger $d\pi/dE$, the larger E^r. Therefore, if we know the properties of $d\pi/dE$, we can discuss the lobbying incentive in more detail. We show a ranking of $d\pi/dE$ at $E = 0$ in Lemma 3, but deriving that for $E \in (0, E^B)$ is quite difficult.

From, the first and second equations in (9), we naturally conjecture that $d\pi^C/dE$ and $d\pi^I/dE$ may be decreasing in E because we assume $e_{xx} < 0$ and $c_{xx} < 0$ and x^C is naturally decreasing in E. From the third equation in (9), we naturally conjecture that $d\pi^T/dE$ may be increasing in E because $d\pi^T/dE$ is proportional to E given $(-dt/dE)$. Therefore, we naturally conjecture that $d\pi^T$ is smaller (larger) than $d\pi^C/dE$ and $d\pi^I/dE$ when E^r is small (large). In such natural situations, we can derive richer implications.

If the initial emission target is small and marginal lobbying cost is high (i.e., the government is tough and thus the lobbying is costly), E^r must be small. In this case, this property suggests that ΔE is the smallest under emission tax policy because $d\pi/dE$ is smallest under emission tax policy. By contrast, if the initial emission target is large or lobbying cost is small (i.e., the government is easily manipulated by the lobbying), E^r must be large. In this case, the emission tax policy yields the largest ΔE because $d\pi/dE$ is largest under emission tax policy. Therefore, if the government has a strong will to implement net-zero emission society, emission tax is desirable, whereas the emission regulations are more reasonable if the government lacks it. Moreover, because $d\pi^T/dE$ is naturally increasing in E, the payoff of the firm can be convex with respect to E when L'' is small. This fact suggests that if the government is fragile to the lobbying and easily manipulated by it (i.e., L' and L'' is small), the corner solution in which $E^r = E^B$ may appear under emission tax policy.

We can show that $d\pi^C/dE$ is decreasing in E as long as e_{qx} and c_{qx} is sufficiently small. However, we fail to derive plausible sufficient conditions under which $d\pi^I/dE$ is decreasing in E because x^I can be increasing in E (Hirose and Matsumura, 2020), which makes the general analysis difficult. Moreover, $(-dt/dE)$ depends on the third-order derivatives of cost and emission cost functions, and thus, we fail to derive plausible sufficient conditions under which $d\pi^T/dE$ is increasing in E.

Although the general analysis of lobbying incentives for $E \in (0, E^B)$ is intractable for the reason mentioned above, parametric analysis in the next section provides the insights on lobbying incentives mentioned above.

4 Parametric Analysis

In what follows, suppose that $P = a - bQ$, $c = \beta q + \gamma x^2/2$, and $e = \kappa q - x$, where a is sufficiently large to ensure the interior solution (i.e., $q > 0$ in equilibrium). In this parametric example, the profit under the emission intensity regulation is greater than that under the emission tax for all $E \in (0, E^B)$

Proposition 2. *$\pi^C(E) > \pi^I(E) > \pi^T(E)$ for all $E \in (0, E^B)$.*

Proof. See Appendix. □

Figure 1 describes Proposition 2 graphically by a numerical example. Because $\pi^C(E^B) = \pi^I(E^B) = \pi^T(E^B)$ for $E = E^B$, Proposition 2 implies that $\pi^C(E^B) - \pi^C(E) < \pi^I(E^B) - \pi^I(E) < \pi^T(E^B) - \pi^T(E)$. Therefore, in contrast to the lobbying incentive in case of zero-emission target, if the initial emission target is loose (i.e., close to the business-as-usual level E^B), an increase from this initial emission target most significantly increases the firm's profit under the emission tax policy, and thus, the emission tax policy is most vulnerable to lobbying.

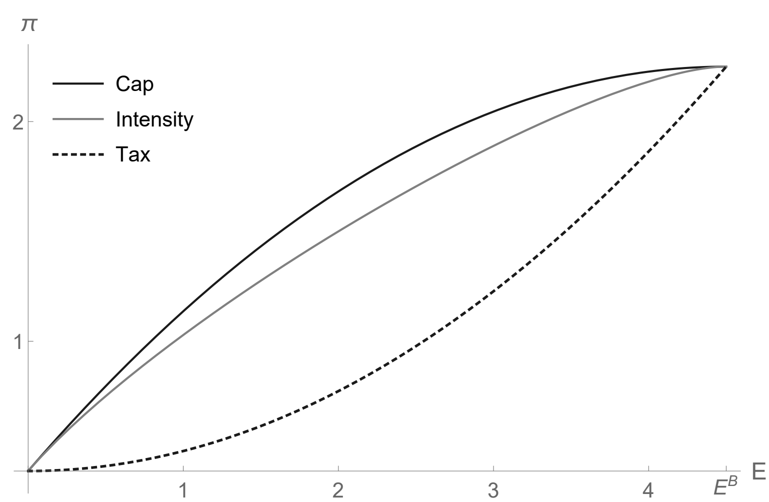

Figure 1: $\pi(E)$ ($a = 5$,$b = 1$,$\beta = 2$,$\gamma = 1$,and $\kappa = 3$)

We now discuss the ranking among $d\pi^i/dE$ for $i = C, I, T$.

Proposition 3. *There exists $\hat{E}_1 \in (0, E^B)$ such that $d\pi^C/dE = d\pi^I/dE = d\pi^T/dE$ when $E = \hat{E}_1$. $d\pi^C/dE > d\pi^I/dE > d\pi^T/dE$ for $E \in (0, \hat{E}_1)$ and $d\pi^C/dE < d\pi^I/dE < d\pi^T/dE$ for $E \in (\hat{E}_1, E^B)$.*

Proof. See Appendix. □

Figure 2 describes Proposition 3 graphically by a numerical example and highlights the regulation robust to lobbying. If the initial emission target E^o is small and marginal lobbying cost L' is high (i.e., the government is tough and thus the lobbying is costly), the realized emission target E^r must be small.[6] In this case, Proposition 3 suggests that ΔE is the smallest under emission tax policy because $d\pi/dE$ is smallest under emission tax policy. Thus, the tax policy is the most robust to polluter lobbying.

By contrast, if the initial emission target is large or lobbying cost is small (i.e., the government is easily manipulated by the lobbying), E^r must be large. In this case, Proposition 3 suggests that the emission tax policy yields the largest ΔE, thus the ta policy is the most vulnerable to polluter lobbying.

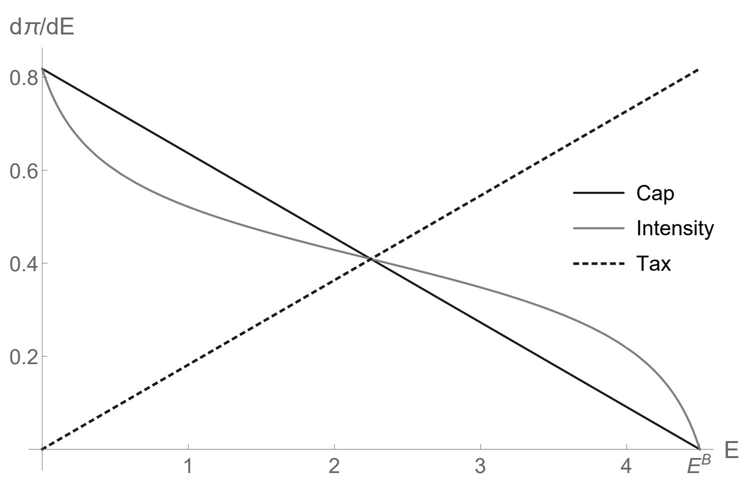

Figure 2: $d\pi/dE$ ($a = 5$, $b = 1$, $\beta = 2$, $\gamma = 1$, and $\kappa = 3$)

[6] For the definitions of E^o, E^r, ΔE, and L, see the third to the last paragraph in Section 3.

We now specify the lobbying cost function and discuss the realized emission target E^r by using numerical examples. Suppose that $L = h(\Delta)^2/2$ where h is positive constant. Figures 3 and 4 show that when h large (small) (i.e., the government is tough (weak) against manipulation by the lobbying and thus lobbying cost is high (low)), the smallest (largest) emission is realized under emission tax policy. Figure 5 shows that when E^o large (small) (i.e., the government is ambitious (adjective) for realizing zero-emission society), the smallest (largest) emission is realized under emission tax policy.

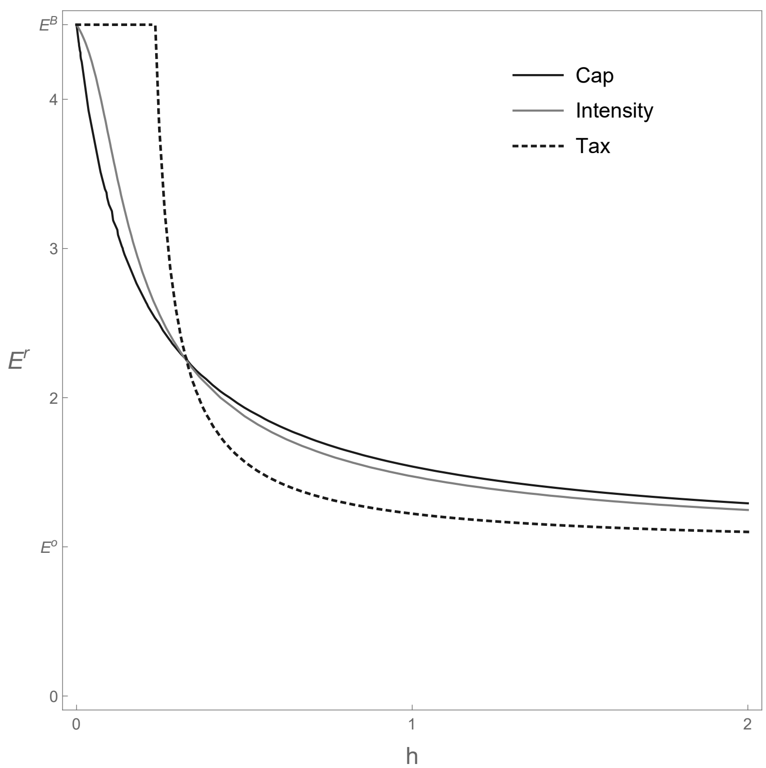

Figure 3: E^r ($a = 5$, $b = 1$, $\beta = 2$, $\gamma = 1$, $\kappa = 3$, and $E^o = 1$)

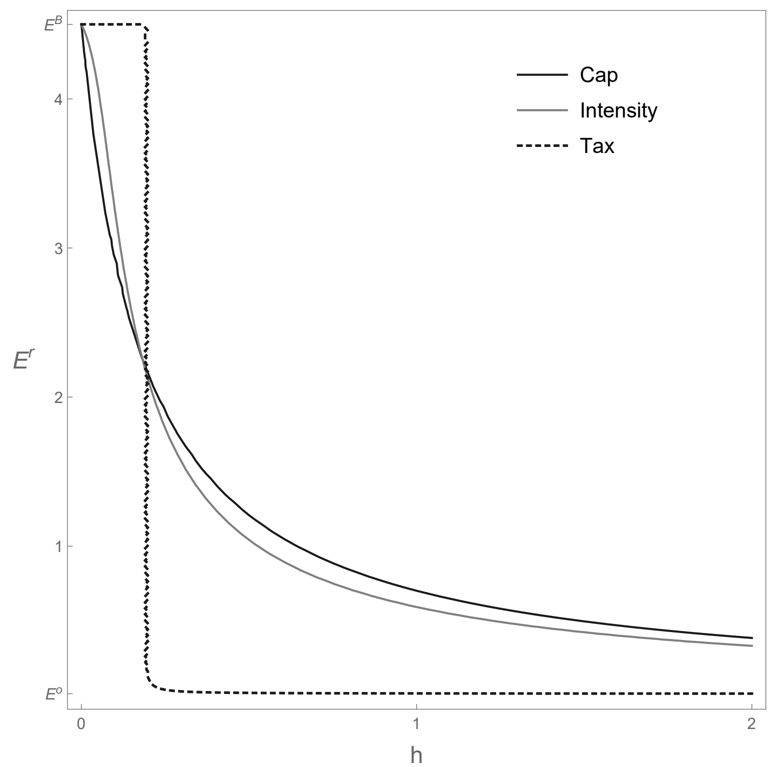

Figure 4: E^r ($a = 5$, $b = 1$, $\beta = 2$, $\gamma = 1$, $\kappa = 3$, and $E^o = 0.01$)

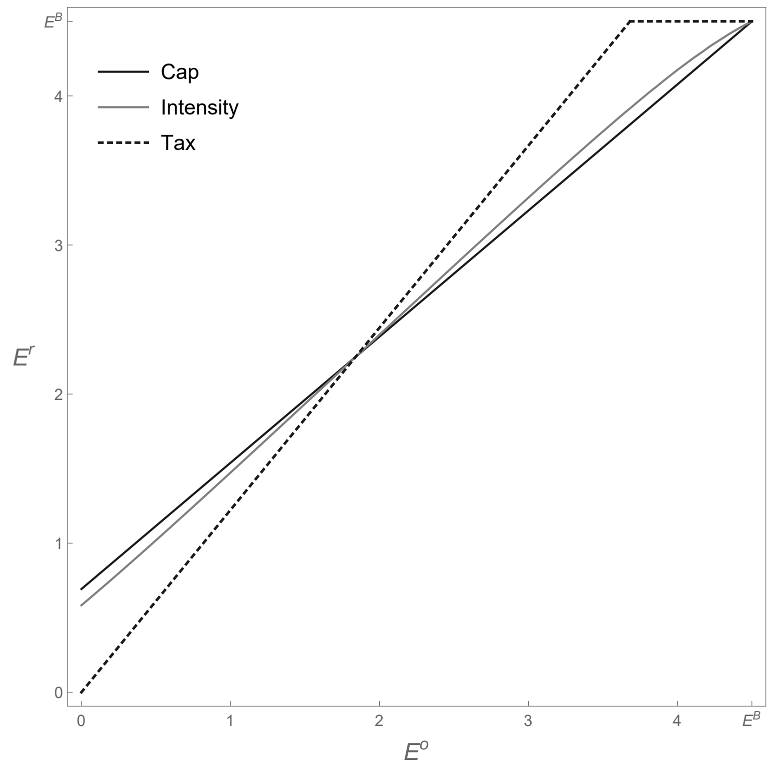

Figure 5: E^r ($a = 5$, $b = 1$, $\beta = 2$, $\gamma = 1$, $\kappa = 3$, and $h = 1$)

5 Conclusion and Policy Implications

In this study, we compare three environmental policies: an emission cap regulation, an emission intensity regulation, and an emission tax. We investigate how the emission target in an industry affects the monopoly firm's profits. We find that when the emission target is close to zero (large), a marginal increase in the emission target yields the least (largest) increase in industry profits under the emission tax. This result implies that among the three policies, the emission tax gives industry leaders the weakest (strongest) incentive to manipulate the emission target when the initial target level is close to zero (large). Therefore, we conclude that the emission tax is a reasonable policy tool when the aim is to achieve a near-zero emission society in the presence of polluter lobbying. We also show that if the government lacks the strong will to implement a near-zero emission society and emission targets can be quite loose, emission regulations can be reasonable policies.

In this study, we mostly focus on the cases in which the emission target is close to zero or close to the business-as-usual level. In both cases, the revenue from the emission tax is small. When the emission target is intermediate, the emission tax revenue could be huge, and firms may lobby to obtain tax refunds rather than to lower the tax rate. Then, the effect on the emission target manipulation reduces, and the government may be able to keep its desirable emissions target even in the presence of polluter lobbying. Incorporating this effect into our analysis remains for future research.

In this study, we consider three environmental policies. Although these policies are popular forms of environmental policies, many other policies, such as energy conservation regulations and green portfolio standards, exist (Holland et al., 2009; Ino and Matsumura, 2021a; Matsumura and Yamagishi, 2017). Moreover, it may be reasonable to combine two or more policies (Cohen and Keiser, 2017; Ino and Matsumura, 2021b). Expanding the range of policy measures is a natural extension of our research.

Appendix

Proof of Lemma 1

(i) Suppose that $E = 0$. Since $\alpha^I q^I = E = 0$ and $q^I > 0$, $\alpha^I = 0$ must hold. Hence, the constraint in Problem (4) is transformed to $e(q, x) \leq 0$, which is equivalent to the constraint in Problem (1). As (4) is identical to (1), $(q^I, x^I) = (q^C, x^C)$.

(ii) q^I is derived from (5). Substituting $q = q^I$ into the left-hand side of equation (5), we have $P + P'q - c_q + c_x(e_q/e_x) = -c_x\alpha < 0$. Because the second-order conditions are satisfied, we have $q^I > q^C$. Because $\alpha^I q^I = E$ and $q^I > q^C$, we have $\alpha^I < E/q^C$.

Proof of Lemma 3

We prove Lemma 3 before Proposition 1.

Suppose $E = 0$. Lemma 1 implies that $\alpha^I = 0$ and $(q^I, x^I) = (q^C, x^C)$. Then, (9) implies $d\pi^C/dE = d\pi^I/dE$. Since both $c_x(q^i, x^i)$ and $e_x(q^i, x^i)$ are positive for $q^i > 0$, $d\pi^C/dE = d\pi^I/dE > 0$. Under the emission tax, since $e(q^T, x^T) = 0$, (9) implies $d\pi^T/dE = 0$.

Proof of Proposition 1

(i) From Lemma 2, we have $\pi^C = p(q^C)q^C - c(q^C, x^C)$ and $\pi^T = p(q^C)q^C - c(q^C, x^C) - t^T e^T$. Because $e^T = 0$ when $E = 0$, we have $\pi^C = \pi^T$ when $E = 0$. When $E = 0$, the maximization problems under the emission cap regulation and emission intensity regulation are exactly the same. Therefore, $q^C = q^I$ and $x^C = x^I$ when $E = 0$, which implies that $\pi^C = \pi^I$ when $E = 0$.

(ii) From Lemma 2, we have $\pi^C = p(q^C)q^C - c(q^C, x^C)$ and $\pi^T = p(q^C)q^C - c(q^C, x^C) - t^T e^T$. Because $e^T = E$ and $t > 0$, we have $\pi^C > \pi^T$ when $E > 0$.

We then show $\pi^C > \pi^I$ when $E > 0$. From Lemma 1, we find that the firm can choose $(q, x) = (q^I, x^I)$ under the emission cap constraint. This implies $\pi^C \geq \pi^I$, and the equality holds only when $(q^C, x^C) = (q^I, x^I)$.

As shown in the proof of Lemma 1, $q^I \neq q^C$ when $\alpha > 0$.

(iii) From Lemma 3, Proposition 1(i), and the continuity of π^I and π^T with respect to E, we have $\pi^I > \pi^T$ when E is sufficiently close to 0.

Proofs of Propositions 2 and 3

Under the parametric assumption, $E^B = (a-\beta)\kappa/2b$. Under the emission cap regulation, given $E \in [0, E^B)$,

$$q^C = \frac{a-\beta+\gamma\kappa E}{2b+\gamma\kappa^2},\ x^C = \frac{(a-\beta)\kappa - 2bE}{2b+\gamma\kappa^2},\ \pi^C = \frac{(a-\beta)^2 - 2b\gamma E^2 + 2(a-\beta)\gamma\kappa E}{2(2b+\gamma\kappa^2)}. \tag{10}$$

Under the emission intensity regulation, given α,

$$q^I = \frac{a-\beta}{2b+\gamma(\kappa-\alpha)^2},\ x^I = \frac{(a-\beta)(\kappa-\alpha)}{2b+\gamma(\kappa-\alpha)^2},\ \pi^I = \frac{(a-\beta)^2}{2(2b+\gamma(\kappa-\alpha)^2)}, \tag{11}$$

and given $E \in [0, E^B)$,

$$\alpha^I = \kappa + \frac{a-\beta-\sqrt{(a-\beta)^2 + 4\gamma E[(a-\beta)\kappa - 2bE]}}{2\gamma E}. \tag{12}$$

Under the emission tax, given t,

$$q^T = \frac{a-\beta-\kappa t}{2b}, x^T = \frac{t}{\gamma}, \pi^T = \frac{2bt^2 + \gamma(a-\beta-\kappa t)^2}{4b\gamma}. \tag{13}$$

and given $E \in [0, E^B)$,

$$t^T = \frac{\gamma[(a-\beta)\kappa - 2bE]}{2b+\gamma\kappa^2}. \tag{14}$$

Substituting (12) and (14) into (11) and (13) respectively, we obtain

$$\pi^I = \frac{(a-\beta)\left(\sqrt{(a-\beta)^2 + 4\gamma E[(a-\beta)\kappa - 2bE]} + a - \beta + 2\gamma\kappa E\right)}{8b+4\gamma\kappa^2},$$
$$\pi^T = \frac{(a-\beta)^2 + 2b\gamma E^2}{4b+2\gamma\kappa^2}.$$

The difference between π^I and π^T is then

$$\pi^I - \pi^T = \frac{(a-\beta)\left\{\sqrt{(a-\beta)^2 + 4\gamma E[(a-\beta)\kappa - 2bE]} - (a-\beta)\right\} + 2\gamma E[(a-\beta)\kappa - 2bE]}{8b + 4\gamma\kappa^2}.$$

Note that since $E^B = (a-\beta)\kappa/2b$, $\sqrt{(a-\beta)^2 + 4\gamma E[(a-\beta)\kappa - 2bE]} \geq a-\beta$ and $(a-\beta)\kappa > 2bE$ for $E \in (0, E^B)$. These yield Proposition 2.

We now prove Proposition 3. Differentiating the profit functions, we obtain

$$\begin{aligned}
\frac{d\pi^C}{dE} &= \frac{\gamma[\kappa(a-\beta) - 2bE]}{2b + \gamma\kappa^2}, \\
\frac{d\pi^I}{dE} &= \frac{\gamma(a-\beta)}{4b + 2\gamma\kappa^2}\left(\frac{\kappa(a-\beta) - 4bE}{\sqrt{(a-\beta)^2 + 4\gamma E[(a-\beta)\kappa - 2bE]}} + \kappa\right), \\
\frac{d\pi^T}{dE} &= \frac{2b\gamma E}{2b + \gamma\kappa^2}.
\end{aligned}$$

Then, the difference between $d\pi^C/dE$ and $d\pi^I/dE$ is

$$\frac{d\pi^C}{dE} - \frac{d\pi^I}{dE} = \frac{\gamma(\kappa(a-\beta) - 4bE)\left(\sqrt{(a-\beta)^2 + 4\gamma E[(a-\beta)\kappa - 2bE]} - a + \beta\right)}{2\left(2b + \gamma\kappa^2\right)\sqrt{(a-\beta)^2 + 4\gamma E[(a-\beta)\kappa - 2bE]}}.$$

Then, $d\pi^C/dE \gtreqless d\pi^I/dE$ if and only if $E \lesseqgtr (a-\beta)\kappa/4b$.

Next, the difference between $d\pi^I/dE$ and $d\pi^T/dE$ is

$$\frac{d\pi^I}{dE} - \frac{d\pi^T}{dE} = \frac{\gamma[\kappa(a-\beta) - 4bE]}{4b + 2\gamma\kappa^2}\left(\frac{a-\beta}{\sqrt{(a-\beta)(a-\beta+4\gamma\kappa E) - 8b\gamma E^2}} + 1\right).$$

Then, $d\pi^I/dE \gtreqless d\pi^T/dE$ if and only if $E \lesseqgtr (a-\beta)\kappa/4b$. Therefore, by defining $\hat{E}_1 = (a-\beta)\kappa/4b$, we obtain Proposition 3.

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Controlling Fake Reviews*

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February 13, 2021

Abstract

This paper theoretically analyzes fake reviews on a platform market using models where a seller creates fake reviews through incentivized transactions, and its sales depend on its rating based on a review history. The platform can control the incentive for fake reviews by changing the parameters of the rating system, such as weights placed on old and new reviews and its filtering policy. At equilibrium, the number of fake reviews increases as quality increases but decreases as reputation improves. Since fake reviews have a positive relationship with a product's underlying quality, rational consumers find a rating more informative when fake reviews exist, while credulous consumers suffer from a bias caused by boosted reputation. A stringent filtering policy can decrease the expected amount of fake reviews and the bias of credulous consumers, but at the same time, it can decrease the informativeness of a rating system for rational consumers. In terms of the weight placed on the review history, rational consumers benefit from higher weights on past reviews than from optimal weights without fake reviews.

[The latest version is available here.]

*"Fully ref**ded after RE**W*
If you are interested dm and comment"

— a post on Facebook

*For continual guidance, I thank my advisor Ichiro Obara. For helpful duscussions, I thank Stepan Aleksenko, Simon Board, Brett Hollenbeck, Akina Ikudo, Akira Ishide, Jacob Kohlhepp, Jay Lu, Toshihiro Matsumura, Moritz Meyer-ter-Vehn, Tomasz Sadzik, Susumu Sato and seminar participants at Kochi University of Technology, UCLA, and University of Tokyo.

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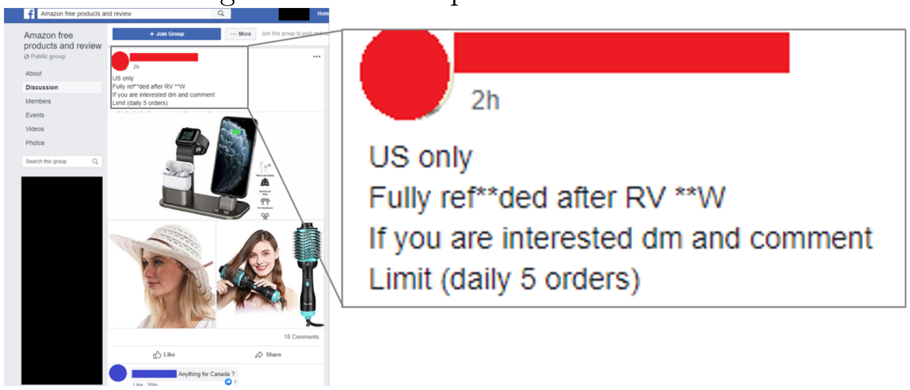
Figure 1: An example of a refund offer



Person Red, who is suspected as a seller on Amazon, posts pictures of its products and offers full refunds of the products after reviews of them. About an hour after of the post, Person Blue, who is suspected as a fake reviewer, shows an interest on the products and refunds.

1 Introduction

Online platform markets are growing worldwide, such that both businesses and their customers increasingly rely on reviews on the platforms.[1] At the same time, incentives for sellers to make fake reviews are also growing. Washington Post (Dwoskin and Timberg, 2018) reports that based on fake review detection algorithms, 50.7% of reviews for Bluetooth headphones, 58.2% for Bluetooth speakers, 55.6% for weight loss pills, and 67.0% for testosterone boosters on Amazon are suspicious. How do sellers make fake reviews? The sellers can post information of their products with refund offers, which are typically finalized via PayPal after purchases and positive reviews on Amazon. (See Fig. 1 for an example of such an offer.) These reviews correspond to verified purchases and are reflected to the star rating (until they are detected by Amazon).[2] He et al. (2020) connect such refund offers on Facebook with product listings on Amazon and show a positive correlation between refund offers on Facebook and a product's performance on Amazon such as its ratings, sales ranking, and the number of reviews. Regulators have been concerned about fake reviews, and their attitude toward fake reviews is becoming stringent. For instance, in 2019, the Federal Trade Commission (FTC) filed the first case against paid fake reviews by CureEncapsulations on Amazon. Online platforms have restricted fake reviews in their own ways, but regulators put increasingly high pressure on online platforms to maintain a stricter attitude against fake

[1] Hollenbeck (2018); Hollenbeck et al. (2019) show that ratings work as a substitute of other form of advertisement or brand names, and this pattern is getting stronger over time in the hotel industry. Reimers and Waldfogel (2020) exhibit that the existence of star ratings has 15 times as the impact on consumer surplus as the professional reviews on New York Times. For the institutional details and data analysis on platforms and ratings, see also Belleflamme and Peitz (2018)

[2] Offers of such fake reviews from fake reviewers have been found on eBay.

reviews. [3]

However, the impact of fake reviews on consumers on a platform is not clear. First, consumers might not be fooled by fake reviews if they know that there are fake reviews. In the standard work of Holmström (1999), the market can correctly anticipate the behavior of long-lived players and debias the signal. Furthermore, customers might be able to elicit additional information from fake reviews. If only high-quality sellers make fake reviews to boost their initial reputation, the boosted rating can be an even better signal of good quality. Such a behavior might be possible if low quality is revealed via word of mouth, and only a high-quality seller can reap benefits from future sales, as suggested by Nelson (1970,1974) in the context of advertising. [4]

In this study, we examine a theoretical model in which sales are determined by the seller's reputation level and the seller chooses the amount of positive fake reviews at each instance. Consumers perceive a seller's reputation based on the potentially boosted ratings displayed on the platform. The platform can control how strictly it filters fake reviews and how much the rating reflects the information of past feedback (i.e., how fast the rating evolves). A key assumption in this study is that it becomes harder for a seller to make fake reviews as its reputation improves because of the higher reimbursement necessary to incentivize reviewers due to the higher price.[5] This brings more fake reviews from the seller with low reputation. This also generates the dependence of fake reviews on the seller's quality-type. Because high-quality sellers benefit more from their high reputation, high-quality sellers generate more fake reviews at equilibrium. Because of this positive relationship between the number of fake reviews and quality, consumers sometimes benefit from lenient policies on fake reviews. In the literature on signaling promotion, the complementarity between quality and reputation is understudied because, in most research, promotion is done only once at the beginning of a game. In this study, the complementarity comes from the future cost-saving effect rather than an increase in revenue.

The opposite dependence of fake reviews on a reputation about quality and on the under-

[3] For instance, in 2019, the Competition and Markets Authority (CMA) in U.K. launched work programme "has written to Facebook and eBay this week urging them to conduct an urgent review of their sites to prevent fake and misleading online reviews from being bought and sold". In responses, both Facebook and eBay have immediately deleted posts identified by CMA, and updated their policy to explicitly prohibit offers of fake reviews. In 2020, May, CMA has launched new investigation into online websites on how they currently detect fake or misleading reviews.

[4] Ananthakrishnan et al. (2020) analyze the display of fake reviews from a different perspective and show that the consumers form more trust on the platform if it shows the fake reviews with flags indicating them as fake reviews, rather than deleting them from the platform.

[5] We can see the interaction between fake reviews and reputation more commonly. For instance, fake reviews might be crowded out if the seller receives many organic feedback due to large demand caused by high reputation. Then, the effective fake review would be costly for such a seller.

lying true quality also provides some cautions on empirical analysis on signaling promotion. That is, reputation-based indices, such as customer rating, can be a bad proxy for a product's underlying quality. Researchers can estimate opposite results if they use customer rating as a proxy for quality. Furthermore, even if the true quality is measured, it is important to control for the reputation level when estimating the relationship between promotion and the underlying quality. Fig. 2 exemplifies the possibility of an omitted variable issue; that is, the promotion level and the true quality of a product can be negatively correlated without being conditioned upon a firm's reputation level, even though quality and promotion have a positive relationship, *ceteris paribus*.

The negative relationship between fake reviews and a firm's reputation also increases the speed at which the rating changes. That is, in the presence of fake reviews, when the rating goes down (up), it more quickly goes up (down) than when the rating system has no fake reviews. This distorts the informativeness of the rating system. How fast the rating changes relates to the relative weight of new information in the rating system. The greater is the weight of new information (and the lower the weight of old information), the faster is the transition of the rating. Thus, the equilibrium effect that makes the transition faster has the effect of distorting upward the weight of the new information (and downward the weight of the old information). Therefore, given the existence of fake reviews, the platform needs to make some adjustments. The platform should set a lower weight for new information (and higher weight for old information) compared with a rating system that has no fake reviews.

The discussion above is based on the assumption of rational consumers who know the seller's strategy. However, the regulator's concern is not necessarily on sophisticated consumers but more on naive consumers, who are vulnerable to fake reviews.[6] In this study, we also incorporate such consumers and show how much they become biased as a result of fake reviews by the sellers. Even though in general the relationship between the bias and the censorship policy is not monotonous, stringent censorship generally reduces the naive consumer's bias under a reasonable range of parameters.

Thus, the regulator might face a trade-off between the precision of the information for rational consumers and the bias that credulous consumers suffer from. This study provides a framework for analyzing such a trade-off.

The remainder of this paper is organized as follows. In Section 2, we review related literature. In Section 3, we analyze a model with rational buyers. In Section 4, we introduce credulous consumers. Section 5 concludes. Most of the proofs are deferred to the Appendix.

[6]For instance, Federal Trade Commission (FTC)'s mission is "[p]rotecting consumers and competition by preventing anticompetitive, deceptive, and unfair business practices through ...". (https://www.ftc.gov/about-ftc)

2 Literature Review

This paper mainly contributes to two streams of literature: rating design and signaling through promotion. The literature on rating design can be divided into two strands: (i) how to reveal the known quality level or estimated quality index (i.e., whether to reveal full information or add noise/coarsen the information) and (ii) how to generate the index of an unknown quality based on the multiple sources of information on a player's performance.

The first strand is often framed in the context of certification, such as the works of Lizzeri (1999), Ostrovsky and Schwarz (2009), Boleslavsky and Cotton (2015), Harbaugh and Rasmusen (2018), Hopenhayn and Saeedi (2019), Hui et al. (2018). Some models are made tractable by the representation with posterior distribution in the line of Bayesian persuasion proliferated by Rayo and Segal (2010) and Kamenica and Gentzkow (2011). Saeedi and Shourideh (2020) extend the framework wherein the quality is endogenously chosen by the seller rather than the exogenous variable.

This paper relates to another strand of literature, as it analyzes how to aggregate the players' actions into a single index. In a one-shot model, Ball (2019) analyzes the optimal way to aggregate the various sources of potentially manipulated signals. In a dynamic setting based on Holmström's (1999) signal jamming/career concern model, Hörner and Lambert (2018) show that the effort level of a long-lived player is maximized by a rating that is linear to past observations. Vellodi (2020) analyzes the impact of rating on the entry/exit behavior of a firm and derives an optimal rating that prevents high-quality sellers from exiting from the market due to a reputation trap of failing to accumulate good reputation because of initial bad luck. Bonatti and Cisternas (2019) examine a long-lived consumer's Ratchet effect. The consumers try to hide its willingness to pay to avoid the personalized pricing by short-lived monopolist, so that the consumption does not perfectly reflect their willingness to pay. Similarly to Hörner and Lambert (2018) and Bonatti and Cisternas (2019), this study examines the relationship between a signal-jamming structure and a linear rating system. In contrast to Hörner and Lambert (2018), the equilibrium strategy is dependent on the hidden quality and reputation, such that the seller's strategy changes the informativeness of the rating on the equilibrium path, as in Bonatti and Cisternas (2019).[7] In contrast to Bonatti and Cisternas (2019), where the effect of the manipulation is endogenously determined via the short-lived player's belief, in this study, the platform controls for the effectiveness of

[7] Another contrast to Hörner and Lambert (2018) is that they start from a general information structure so that they can represent any reputation by changing the information structure. Then, they can focus on the resulted process of reputation level in a similar way that researchers focus on the resulted outcome by the revelation principle in the context of the mechanism design. On the other hand, this paper and Bonatti and Cisternas (2019) use more specific information structure, so that we should examine how the consumers interpret the resulted rating.

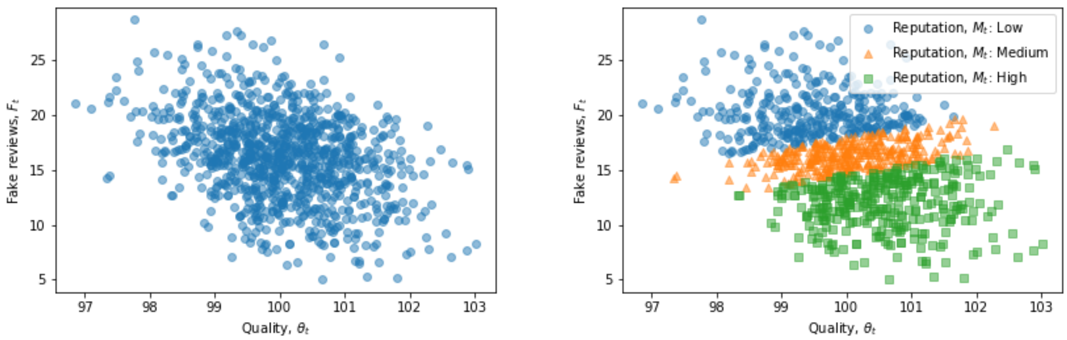
the manipulation so that we can analyze the impact of censorship by the platform. In addition, this study departs from the literature by analyzing the impact of manipulation on naive/credulous consumers, which is often the concern of regulators.

This paper also contributes to the literature on promotion and signaling. Nelson (1970, 1974) argues that even if the promotion does not have any intrinsic information, "burning money" itself can be a signal of good quality because such a signal pays off only for high-quality firms through repeated purchases in the future. This idea is formalized later by Kihlstrom and Riordan (1984), Milgrom and Roberts (1986a) and many others as separating equilibria in signaling models. Using a one-shot signal-jamming framework instead of a signaling model, Mayzlin (2006) shows a negative relationship between promotion through fake reviews and quality, and Dellarocas (2006) generalizes conditions for the positive/negative correlation in a one-shot signal-jamming model. Bar-isaac and Deb (2014) examine the effects of vertically/horizontally heterogeneous preferences, and Grunewald and Kräkel (2017) examine the effect of competition between firms. Most studies on the signaling role of promotion are based on models with one-shot promotion, except for Horstmann and MacDonald (1994), where the experience of the product is an imperfect signal of the quality, and the signaling via advertising is done only after establishing a reputation so that it is hard for low-quality sellers to mimic high-quality sellers' behavior.[8]. In this study, I examine a dynamic signal-jamming model, where reputational concern is the driving force for the positive correlation between quality and promotion. It also generates non-degenerate dynamics consistent with an observation by Luca and Zervas (2016) that strategic manipulation increases after a drop in reputation.

The dependence of fake reviews on reputation also provides some implications for the empirical literature on signaling promotion. The literature has had weak support regarding the correlation between quality and promotion. For instance, Kwoka (1984) observes that optometrists with more advertisements provide less thorough eye examination, and Horstmann and Moorthy (2003) observe that advertising is hump-shaped in terms of quality among restaurants in New York. Recently, Sahni and Nair (2019) implement a quasi-experiment to isolate the intrinsic information and signaling effect of burning money and show that the consumer positively responds to the burning of money. They point out that it is difficult to show the relationship between quality and promotion level because it is difficult to obtain a reliable measure of quality. This paper emphasizes this point. A reputation-based index, such as customer rating, can be a bad proxy for the underlying quality. The reputation

[8]Aside from the context of the rating system or the signaling promotion, Grugov and Troya-Martinez (2019) examine the biasing behavior of the seller in a model a. la. Holmström (1999) incorporating a detection rule and credulous consumers, and show that the biasing behavior increases as the authority requires stricter rule and the share of credulous consumer increases.

Figure 2: A simulated distribution of quality levels and the amount of fake reviews



The left panel show that the the amount of the fake reviews is negatively correlated with the quality level, unconditional on the level of reputation. On the other hand, the right panel shows that the amount of the fake reviews is increasing in the quality level, conditional on the reputation level.

level and the underlying quality level have opposite impacts on the promotion level in equilibrium. Furthermore, even if the true quality is measured, it is important to control for the reputation level. As shown in Fig. 2 , the level of promotion and the true quality can be negatively correlated without being conditioned upon the reputation level, even though quality and promotion have a positive relationship, *ceteris paribus*.

3 Rating Design for Rational Consumers

In this study, we examine both models with rational consumers and naive consumers. In this section, we first introduce a baseline model with a mass of rational consumers. The consumers rationally expect that a long-lived seller makes fake reviews following a linear strategy. However, they cannot induce the seller's exact action at time t because the quality is still hidden, even though the strategy and the current reputation are known to the consumers.

Then, in the next section, we introduce a market with naive consumers who do not expect any fake reviews while the seller makes fake reviews, such that the reputation is biased upward. In each model, we examine the impact of the platform's filtering/censoring policy on reviews, the weights of new and old reviews, and the precision of genuine reviews.

3.1 Model

The model is in a continuous time and infinite horizon, $t \in [0, \infty)$. At each instance t, a long-lived seller sells q units of its product, whose quality is denoted as θ_t, and makes F_t units of fake reviews. A sufficiently large mass, n, of consumers forms a demand function such that the price $p_t = E[\theta_t|Y_t] \equiv M_t$ clears the market, where Y_t is the rating of the product at time t.[9] The price being a representation of the reputation of the hidden quality is the standard assumption in the literature on reputation. The quality θ_t governs consumers' willingness to pay for the product, so the price is high when the expected quality of the product is high. A more specific underlying model, that can incorporate naive consumers is suggested in the Appendix.

The quality, θ_t, and rating, Y_t, change over time. The quality, θ_t, follows an exogenous mean-reverting process:

$$d\theta_t = \kappa\left(-\theta_t + \mu\right)dt + \sigma_\theta dZ_t^\theta \tag{1}$$

while the rating, Y_t, is characterized by the following differential equation:

$$dY_t = -\phi Y_t + d\xi_t \tag{2}$$

where $d\xi_t$ is defined as:

$$d\xi_t = aF_t dt + bq\theta_t dt + \sqrt{bq}\sigma_\xi dZ_t^\xi \tag{3}$$

where $\left(Z_t^\theta, Z_t^\xi\right)$ is a standard Brownian motion; a is the effectiveness of the fake review; b is the feedback rate from customers; μ is the mean of θ_t in the stationary distribution, and σ_θ and σ_ξ govern the standard deviations of the disturbance. The exogenous mean-reverting process of θ_t is understood as resulting from the competition over quality among sellers. The relative quality of a firm's product might decrease due to the rise of other sellers with even higher quality. The firm's product's relative quality might increase when a competitor increases its product's price. The transition of the rating, Y_t, is interpreted in a discrete time analogue that the future rating, Y_{t+dt}, is a weighted sum of the new reviews, $d\xi_t$, and the previous reviews, Y_t, with weights of 1 and $1 - \phi dt$, respectively. After filtering suspicious reviews, the new reviews consist of two components: "organic" reviews and the remaining fake reviews. The second and third terms of Eq. (3) correspond to organic reviews. Higher quality tends to generate high reviews, and the information becomes precise when there is feedback from many transactions (i.e., high q) or a high response rate (i.e., high b). The disturbance, $\sigma_\xi dZ_t^\xi$, is caused by the heterogeneity of the criteria among customers.[10] The first term is the effect of the fake reviews. The seller tries to boost the average review through fake reviews, but some of them are detected by the platform, and the remaining reviews enter as $aF_t dt$. Thus, a small a implies stringent censorship. As in Hörner and Lambert (2018),

[9]Saeedi (2019) showed that the reputation is the measure determinant of the price on eBay market.

[10]In this paper, the mechanism behind the customer feedback is abstracted and assumed that the fixed portion of consumers keep reviewing. For detailed analysis on the customer feedback, see Chevalier et al. (2018) and the literature cited in it. They analyze the relationship with managerial responses to reviews.

Vellodi (2020), and Bonatti and Cisternas (2019), the rating, Y_t, does not exactly capture 5-star rating on Amazon, Yelp, or some other online platform. The level of Y_t is dependent on the mean of θ_t and other parameters. By this specification of the rating, we can rely on the normality to simplify the analysis.

The seller's instantaneous payoff is defined as:

$$\pi_t = (1-\tau)\, p_t\,(q+F_t) - p_t \cdot F_t - \frac{c}{2}F_t^2$$

where τ denotes the transaction fees imposed by the platform. The first term is the total revenue from all transactions, including those corresponding to fake reviews, and the second term is the reimbursement cost to the fake reviewers. The last term expresses that generating more fake reviews is harder. The seller might find it challenging to search for incentivized reviewers through communities such as Facebook. Some fake review services may charge a higher price for fake reviews. Furthermore, increasing the number of fake reviews come with a higher risk of being detected by the platform. The cost of production is abstracted out from the model. [11] The long-lived seller maximizes its discounted present value by choosing $(F_t)_{t\geq 0}$.

The instantaneous profit becomes easier to compare with the previous research when it is rewritten as follows:

$$\pi_t = (1-\tau)\, M_t \cdot q - \tau M_t \cdot F_t - \frac{c}{2}F_t^2. \tag{4}$$

Without the second term in eq. (4), the model becomes effectively a special case of Hörner and Lambert (2018), which is based on Holmström's (1999) signal-jamming model and uses a general information structure as a rating. However, due to the existence of this term, the marginal cost of the manipulation depends on the current reputation level. Therefore, the equilibrium manipulation level depends on the current rating in contrast to Hörner and Lambert (2018), where the equilibrium action turns out to be state-independent. Instead of relying on the time- and state-invariant action, we apply the idea of Bonatti and Cisternas (2019) to focus on a linear strategy, and a Gaussian stationary distribution of (θ_t, Y_t). Then, the Hamilton-Jacobi-Bellman equation gives a simple quadratic value function, which is solved by the guess-and-verify method. It is verified that as τ approaches zero, the equilibrium strategy becomes invariant to θ_t, Y_t, (and t).

[11] Whether the high quality seller or low quality seller face high costs of production is arguable by itself. If high quality come from the seller's high productivity, the high quality seller can produce with lower costs. If the low quality is by the seller's choice rather than the difference in the production technology among sellers, the low quality product would be associated with low production cost. The different specifications on the production costs can cause different pattern in fake reviews, but those extensions are deferred to the future research.

The interaction between the current reputation and the current action is considered as the driving force of the non-degenerate Markov equilibrium strategy. In this study, this interaction between reputation and manipulation is derived from the reimbursement to fake reviewers; however, such an interaction can be more commonly observed in the context of fake reviews. For instance, if the reputation is high, then a large demand can crowd-out fake reviews, such that the effective fake reviews can be more costly given the high reputation. In the Appendix, an alternative model with such an interpretation is discussed. A model with a changing quantity that is isomorphic to the main model is discussed in Appendix C.

Definition of the Equilibrium As mentioned above, we focus on a linear Markov strategy equilibria, where a linear Markov strategy maximizes the seller's discounted present value among any admissible strategies.

A linear strategy (in θ_t and Y_t) is defined as:

$$F_t = \hat{\alpha}\theta_t + \hat{\beta}Y_t + \hat{\gamma}$$

Note that θ_t does not directly appear in the instantaneous payoff function, but it appears in the transition of the payoff relevant state variable, Y_t. Thus, the seller is potentially sensitive to the level of θ_t. Now the equilibrium is defined as follows:

Definition 1. A linear Markov strategy $F = (F_t)_{t\geq 0}$ s.t. $F_t = \hat{\alpha}\theta_t + \hat{\beta}Y_t + \hat{\gamma}$ is a stationary Gaussian linear Markov equilibrium if

1. $F = \arg\max_{(\tilde{F}_t)_{t\geq 0}} E_0\left[\int_0^\infty e^{-tr}\pi_t\right]$ where $\left(\tilde{F}_t\right)_{t\geq 0}$ is admissible,
2. $M_t = E\left[\theta_t | Y_t\right]$, and
3. $(\theta_t,\, Y_t)_{t\geq 0}$ induced by F is a stationary Gaussian.

We do not know that $(\theta_t,\, Y_t)_{t\geq 0}$ is stationary or Gaussian *ex ante* because Y_t is endogenously determined by F_t. However, given a linear strategy, the condition for $(\theta_t,\, Y_t)_{t\geq 0}$ to be a stationary Gaussian is simply characterized by an inequality—similar to Bonatti and Cisternas (2019)—by Eqs. (2) and (3), and the definition of the linear strategy,

$$\begin{aligned} dY_t &= -\phi Y_t dt + aF_t dt + bq\theta_t dt + \sqrt{bq}\sigma_\xi dZ_t^\xi \\ &= -\left(\phi - a\hat{\beta}\right) Y_t dt + \left(a\hat{\alpha} + bq\right)\theta_t dt + a\delta\mu dt + \sqrt{bq}\sigma_\xi dZ_t^\xi \end{aligned} \tag{5}$$

Thus, an inequality, $\phi - a\hat{\beta} > 0$, must hold for $(\theta_t,\, Y_t)_{t\geq 0}$ to have a stationary distribution (otherwise, the process of Y_t diverges). When $(\theta_t,\, Y_t)$ is a stationary Gaussian, by the projection theorem on the Gaussian distribution,

$$M_t \equiv E\left[\theta_t | Y_t\right] = E\left[\theta_t\right] + \frac{Cov\left(\theta_t,\, Y_t\right)}{Var\left(Y_t\right)}\left[Y_t - E\left[Y_t\right]\right] \tag{6}$$

Furthermore, if it is stationary, all expectations in Eq.(6) are constants. By letting $\lambda \equiv \frac{Cov(\theta_t, Y_t)}{Var(Y_t)}$ and $\nu \equiv E\left[Y_t\right]$ (and $\mu = E\left[\theta_t\right]$ by construction), Eq.(6) is written as $M_t = \mu + \lambda[Y_t - \nu]$. In the following part of this section, we use M_t instead of Y_t as a state variable for the sake of expositional simplicity. Then, the linear strategy is redefined as

$$F_t = \alpha\theta_t + \beta M_t + \delta\mu$$

The stationary condition is summarized as follows:

Lemma 1. *(Stationarity and the characterization of the long-run moments) Suppose $F_t = \alpha\theta_t + \beta M_t + \delta\mu$ where $M_t \equiv E\left[\theta_t|Y_t\right]$ for all $t \geq 0$. Then, a process $(\theta_t,\, Y_t)_{t\geq 0}$ is a stationary Gaussian if and only if*

i. $M_t = \mu + \lambda\left[Y_t - \nu\right]$ *for all* t

ii. $a\lambda\beta - \phi < 0$, *and*

iii. $(\theta_0,\, Y_0)' \sim \mathcal{N}\left([\mu,\, \nu]',\, \Gamma\right)$ *is independent of* $\left(Z_t^\theta,\, Z_t^\xi\right)_{t\geq 0}$ *where* Γ *is the variance-covariance matrix in the stationary distribution.*

The third condition is required so that the game starts from a stationary distribution. Now, the HJB equation is simply written by using Ito's lemma:

$$\begin{aligned} rV\left(\theta,\, M\right) = &\sup_{F\in\mathbb{R}} \left(1-\tau\right) M \cdot q - \tau M \cdot F - \frac{c}{2}F^2 \\ &- \kappa\left(\theta - \mu\right) V_\theta \\ &+ \left\{a\lambda F + bq\lambda\theta - \phi\left[M - \bar{\theta} + \lambda\bar{Y}\right]\right\} V_M \\ &+ \frac{\sigma_\theta^2}{2} V_{\theta\theta} \\ &+ \frac{bq\lambda^2\sigma_\xi^2}{2} V_{MM} \end{aligned} \tag{7}$$

By guessing the quadratic form of the value function, $V = v_0 + v_1\theta + v_2 M + v_3\theta^2 + v_4 M^2 + v_5\theta M$, and the linear strategy, we can verify the existence and uniqueness of the value function and the linear strategy via the matching coefficient.

3.2 Equilibrium Characterization

The equilibrium strategy is characterized by guessing the quadratic value function and the linear strategy and by matching coefficients α, β, δ, $(v_k)_{k=0}^{5}$ of the first-order conditions, envelop conditions, and the stationarity condition characterized in Lemma 1. In the proof, the characterizing conditions are summarized into one equation $h(L) = 0$ with an aggregator $L \equiv a\lambda\beta$, and then all the equilibrium coefficients are derived as a function of L. Aggregator L is interpreted as an equilibrium effect on the speed of the rating transition or the equilibrium effect on the relative weight of new information. When L is positive, the rating transition effectively speeds up because the low rating is soon boosted back to the average rating by fake reviews.

By analyzing the existence and uniqueness of the aggregator L and examining the corresponding equilibrium coefficients, we obtain the following theorem:

Theorem 1 (Existence and uniqueness). *There is always a stationary linear Markov equilibrium. For any equilibrium,* $\alpha > 0$, $\beta \in \left(-\frac{\tau}{c}, 0\right)$, $\lambda > 0$ *and* $L > 0$ *hold. Furthermore, if* $h'(L) < 0$ *holds, then such an equilibrium is unique, and the equilibrium coefficients* α, β, *and* δ *are differentiable in the parameters.*

$h'(L) < 0$ *holds for any* $L > 0$ *if* $6\kappa\phi + 4r^2 + 2\kappa r + 17r\phi + 19\phi^2 > \kappa^2$.

Note that $6\kappa\phi + 4r^2 + 2\kappa r + 17r\phi + 19\phi^2 > \kappa^2$ is a loose and reasonable condition. ϕ is the transition speed of the rating, and κ is the transition speed of the quality. The required inequality is reasonable as long as the rating system is meant to help estimate the current quality. For instance, even if the true quality does not drift much (i.e., $\kappa \simeq 0$), the rating should drift toward the underlying true quality (i.e., $\phi > 0$).

Intuition of the Equilibrium Strategy In Theorem 1, it is shown that high-quality types make more fake reviews ($\alpha > 0$), conditional on its reputation level. and high-reputation type makes fewer fake reviews ($\beta < 0$) conditional on the quality type. Given the logic of Nelson (1970; 1074), $\alpha > 0$ (and $\beta < 0$) might look intuitive, but this model adds different reasons than the previous research.

I start from the negative β. From the first-order condition, the optimal strategy is expressed as

$$F_t = -\frac{\tau}{c}M + a\lambda \underbrace{\{v_2 + 2M_t v_4 + \theta v_5\}}_{=V_M}$$

Then, $\beta = -\frac{\tau}{c} + \frac{2a\lambda}{c}v_4$. Furthermore, the envelope condition gives an expression for v_4 so that it is rewritten as $\beta = -\frac{\tau}{c} - \frac{\tau}{c}\frac{a\beta\lambda}{(-a\beta\lambda + r + 2\phi)}$. The first term comes from the interaction of the reputation level and the fake reviews in the cost term, $\tau M_t F_t$. If the reputation is high, then the marginal cost of the fake review is high. Therefore, the seller will make fewer fake reviews given a higher reputation. The second term corresponds to the fake review's marginal benefit in the future. Given the equilibrium strategy, $v_4 = -\frac{\beta\tau}{2(-a\beta\lambda+r+2\phi)}$ is positive, meaning that the marginal benefit in the future increases with the reputation. This is because the future self will reduce the amount of fake reviews after observing the boosted reputation due to today's fake reviews. Furthermore, this effect increases with M_t because the future reputation M_{t+dt} tends to be high given a high M_t, so the interaction term

$$\tau M_{t+dt} F_{t+dt} = \alpha\tau M_{t+dt}\theta_{t+dt} + \tau\beta M_{t+dt}^2 + \delta\mu\tau M_{t+dt} \tag{8}$$

decreases quadratically given a negative β. It turns out that the first term dominates the second term; thus β remains negative.

The intuition of positive α comes from the complementarity between the quality, θ, and the reputation, M, in the seller's value function. With high quality θ_t today, the reputation in the future tends to be higher than the case with low quality today, given the same level of reputation M_t today. Furthermore, as previously stated, the future benefit from the reputation boost is higher given a higher reputation in the future. Thus, high quality results in a high incentive for fake reviews. Mathematically, the equilibrium coefficient α is characterized as

$$\alpha = a\lambda v_5 = \frac{a\lambda}{\kappa + r + \phi}\{2\left(a\alpha + bq\right)\lambda v_4 - \alpha\tau\} \tag{9}$$

The first equality reveals that the sign of α comes from the complementarity of θ and M in the value function. In the last expression, $(a\alpha + bq)\,\lambda$ indicate that the high θ_t results in a high M_{t+dt}. It is multiplied with positive v_4, which represents an increasing marginal value with respect to M_{t+dt}. This is the driving force of the positive α. The remaining term of Eq. (9), $-\alpha\tau$, states that such an incentive is attenuated because the quality in the near future θ_{t+dt} tends to be high given high θ_t; thus, today's fake reviews increase the cost in the future via the first term of Eq. (8).

In summary, the driving force of $\beta < 0$ is the incentive to reduce $\tau M_t F_t$ today given a high M_t. α is positive because of the complementarity of θ_t and M_t through cost savings. Readers might wonder why an increase in revenue (like Nelson, 1970, 1974) does not appear in the above argument. If θ_t is high, the boosted revenue would stay high for a long time; but in this model, such a product would eventually achieve a high reputation through organic feedback even without fake reviews. Therefore, the *marginal future revenue* $\frac{dp_s}{dF_t}$ $(s \geq t)$ is independent of θ_t. It is worth noting that the same intuition applies even in a variant of the model with a fixed price p and time-varying quantity q_t discussed in the Appendix.

3.2.1 Properties of the equilibrium

Before examining the normative properties of the equilibrium, we check some positive properties of the equilibrium.

First, the expected amount of fake reviews is increasing in a. This is simply because the marginal benefit of fake reviews in the future would increase if the platform loosens the censorship policy. The model does not guarantee a positive amount of fake reviews in general, but it is also shown that the expected amount of fake reviews is positive under some parameters.

Proposition 1. *$E[F_t]$ increases with L and L increases with a. Furthermore, $E[F_t] \geq 0$ holds for sufficiently large a.*

Thus, the model can represent a reasonable situation under some parameters where fake reviews have non-trivial effect (i.e., a is significantly high). There still remains a small probability that F_t becomes negative due to the normal distribution, but the model can approximate a reasonable distribution of the fake reviews, under which the negative revenue is rarely observed, as shown in Fig. 2.

The precision of "organic" feedback from normal customers also monotonically changes the expected amount of fake reviews. When the organic feedback from customers varies a lot, it is hard for the seller to manipulate the reputation because a boosted rating is attributed to a large variation in the feedback.

Proposition 2. *$E[F_t]$ is decreasing in $\left(\frac{\sigma_\xi}{\sigma_\theta}\right)$.*

Even though a stringent policy decreases the expected amount of fake reviews, as shown in Proposition 1, it does not imply that the seller's strategy gets closer to the no-fake strategy of $\{\alpha, \beta, \delta\} = \{0, , 0, 0\}$. Moreover, the stringent policy might have unintentional effects of increasing the absolute value of the equilibrium coefficients.

Proposition 3. *$|\alpha|$ increases in $\frac{\tau}{c}$ and decreases in $\frac{\sigma_\xi}{\sigma_\theta}$. $|\beta|$ decreases in a and increases in $\left(\frac{\sigma_\xi}{\sigma_\theta}\right)$.*

Under a stringent policy (small a), the marginal benefit of fake review decreases because fake reviews are reflected less in the rating; but at the same time, the dependence of the marginal benefit on the current reputation also decreases. Mathematically, the second term of $\beta = -\frac{\tau}{c} + \frac{\tau}{c}\frac{-a\beta\lambda}{(-a\beta\lambda+r+2\phi)}$ decreases while the marginal cost still depends on the current reputation regardless of the censoring policy. Therefore, $|\beta|$ increases owing to the less countervailing effect.

In the proof of the proposition, the intensity of dynamic consideration is also captured by an aggregator $L = -a\lambda\beta$, which is the equilibrium effect on the reputation transition speed. L becomes smaller when the dynamic incentive becomes smaller; thus, α, which only comes from the future marginal benefit, becomes smaller, and $|\beta|$, to which the future marginal benefit only works as a counteracting effect, becomes greater because the present cost reduction incentive prevails. L is shown to be increasing in $\frac{a\tau}{c}$ and decreasing in $\frac{\sigma_\xi}{\sigma_\theta}$.

Lemma 2. *L at the equilibrium increases in $\frac{a\tau}{c}$ and decreases in $\frac{\sigma_\xi}{\sigma_\theta}$. Furthermore, $L \to 0$ as $\frac{a\tau}{c} \to 0$ and $L \to \infty$ as $\frac{a\tau}{c} \to \infty$.*

This concludes Proposition 3. α does not necessarily increase in a because α is a function in a and L, so the change in a affects directly and indirectly via L, and the net impact is not clear. $|\beta|$ does not necessarily decrease in $\frac{\tau}{c}$ for an analogous reason even though a limit of $\tau \to 0$ is known.

Proposition 3 implies less signaling (smaller α) and more distortion in the effective transition speed of the rating (greater $|\beta|$) when the aggregator on the strategic effect L is small. This suggests less information from the rating system when the strategic effect L is small. In the following section, we formally examine this effect.

Some limits of the equilibrium strategy are worth noting before jumping into a normative analysis. Since the negative β comes from the interaction term in the cost of the fake reviews, whose coefficient is τ, β approaches zero as τ approaches zero. At the same time, α also approaches zero because the complementarity of θ and M is caused by future cost savings via negative β. In this limit, the fake reviews become constant as in Holmström (1999). This is summarized in the following proposition.

Proposition 4. *$|\alpha|, |\beta| \to 0$ as $\tau \to 0$.*

3.3 Optimal Rating System for Rational Consumers

In this study, we focus on the informativeness of the rating system as a normative criterion for two reasons. First is from the viewpoint of consumer protection: as the rating system gets more informative about the quality of a product, the price is likely to be close to the underlying quality. Thus, it becomes less likely that consumers would face huge regret from the purchase of the product. Second is from the viewpoint of the platform: the informativeness of the rating is crucial to attracting consumers in the long run. If consumers find it uninformative, they, as well as the sellers, can move to other platforms, given less consumers in the market. Thus, the informativeness of the rating would be the first priority when the platform controls it.

Since rational customers can form an unbiased estimate from any current rating, $M_t = E[\theta_t|Y_t]$, the informativeness of the rating is defined by the variance of the customer's estimate of quality. Owing to the normality assumption, this is rewritten as $Var(\theta_t|Y_t) = Var(\theta_t)(1-\rho^2)$, where ρ^2 is the correlation between θ_t and Y_t. Therefore, we use ρ^2 as the criterion for the informativeness of the rating.

Given an equilibrium strategy, the stochastic differential equations—Eqs. (1) and (5)—give us ρ^2 as a function of the parameters and the equilibrium strategy. Therefore, the change of a parameter directly affects ρ^2 and indirectly affects it via a change of the equilibrium strategy. Fortunately, by representing the equilibrium coefficients α and β as functions of the equilibrium aggregator $L = a\beta\lambda$, all the direct and indirect effects of the censorship (a) are expressed as an effect through L. Comparative statics about other parameters, such as ϕ and σ_ξ/σ_θ, can also be examined by the indirect effect through L and the direct effect.

Lemma 3. *At the equilibrium, ρ^2 is expressed as a function:*

$$\rho^2(L;\, \phi,\, \kappa,\, \sigma_\xi,\, \sigma_\theta,\, r,\, bq) = \frac{(\phi + L)}{(\kappa + \phi + L)} \frac{(A(L;\, \phi,\, \kappa,\, r,\, bq) + 1)^2}{((A(L;\, \phi,\, \kappa,\, r,\, bq) + 1)^2 + \kappa(\sigma_\xi/\sigma_\theta)^2(\kappa + \phi L))}$$

on which a, c, τ have effects only through L.

$A(L;\, \phi,\, \kappa,\, r,\, bq)$ summarizes all the direct and indirect effects of a on the informativeness as a function of L.

3.3.1 Filtering/Censoring Reviews

First, we analyze the impact of a filtering/censoring policy, a. Do fake reviews damage the informativeness of the rating system compared with the case without fake reviews? Does filtering or censoring the reviews (i.e., decrease in a) increase the rating's informativeness?

As a benchmark, we derive informativeness *without* fake reviews. By construction, we can do this by letting $\alpha = \beta = \delta = 0$.[12] The same informativeness is also replicated by setting $L = 0$ in $\rho^2(L;\, \phi,\, \kappa,\, \sigma_\xi,\, \sigma_\theta)$ to make it easier to compare with the informativeness at the equilibrium.

Lemma 4. *$\rho^2(0;\, \phi,\, \kappa,\, \sigma_\xi,\, \sigma_\theta)$ coincides with ρ^2 under the no-fake strategy.*

Note that $L = 0$ does not necessarily mean $\alpha = \beta = \delta = 0$. For instance, L approaches 0 as a approaches 0; but at the same time, β converges to some negative value. The lemma says that even under such a situation, informativeness is the same as that without fake

[12] Actually, δ does not enter in the formula for the informativeness, so $\delta = 0$ does not matter in terms of the informativeness.

reviews. Lemma 2, which is about the relationship between L and parameters, and Lemma 4 together lead us to the following proposition:

Proposition 5. *The informativeness of the rating system in equilibrium converges to that of the "no-fake" strategy as* $\frac{a\tau}{c} \to 0$.

Thus, even though the equilibrium strategy at the limit of $\frac{a\tau}{c}$ is not necessarily the no-fake strategy, the informativeness converges to that of the no-fake strategy.

By analyzing the behavior of $\rho^2 (L; \phi, \kappa, \sigma_\xi, \sigma_\theta)$ with respect to L, we can conclude that the informativeness can be even higher under some parameters where a positive amount of the fake reviews is expected. In other words, stringent censorship can decrease the informativeness of the rating system.

Proposition 6. *The equilibrium strategy is more informative than no-fake strategy under a set of parameters such that*

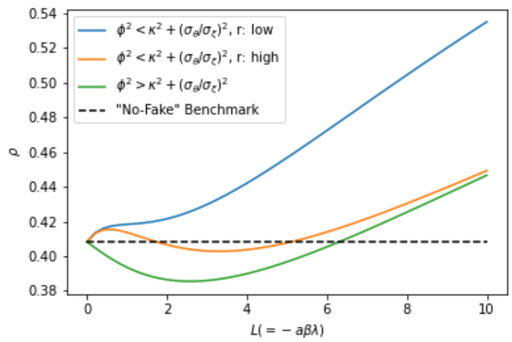
1. $\frac{a\tau}{c}$ *is sufficiently large, or*
2. $\frac{a\tau}{c}$ *is sufficiently small and* $\phi^2 < \kappa^2 + \frac{\sigma_\theta^2}{\sigma_\xi^2}$

Fig. 3 shows the behavior of ρ^2 with respect to L. The first part of the proposition comes from the fact that ρ^2 converges to 1 as L approaches infinity. Since L is increasing $\frac{a\tau}{c}$ from zero to infinity, the equilibrium informativeness surpasses that of the no-fake benchmark at some point as $\frac{a\tau}{c}$ increases. The second part is derived from the behavior of ρ^2 around $L = 0$. The derivative of ρ^2 with respect to L is determined by the relative size of ϕ^2 and $(\kappa^2 + \sigma_\theta^2/\sigma_\xi^2)$: If $\phi^2 < \kappa^2 + \frac{\sigma_\theta^2}{\sigma_\xi^2}$, then ρ^2 decreases in L; thus, decreases in $\frac{a\tau}{c}$. [13]

The intuition of this proposition consists of two parts: (i) As mentioned in Subsection 3.2.1, the sensitivity of fake reviews to θ_t decreases as the strategic effect L decreases. Thus, the strict censorship policy, which reduces the equilibrium aggregator L, decreases the signaling effect of the fake reviews. (ii) Meanwhile, $L > 0$ increases the effective transition speed of reputation to $\phi + L$. It can be good or bad in terms of informativeness, depending on the original transition speed, ϕ. More specifically, the threshold of $\sqrt{\kappa^2 + \sigma_\theta^2/\sigma_\xi^2} \equiv \phi^0$ is the informativeness-maximizing ϕ, given no fake reviews. Therefore, if ϕ is smaller than ϕ^0, the faster transition improves informativeness. It turns out that the first effect dominates in the case of a large L and the second effect dominates in the case of L close to zero.

[13] Note that $E[F_t]$ is increasing in L and positive for large L (by Proposition 1). Thus, the high informativeness is not due to negative fake reviews, but associated with the positive amount of fake reviews.

Figure 3: Change of the informativeness in the aggregator L



The graph indicates that the informativeness is (i) increasing in L if ϕ and r are relatively low, (ii) increasing in L around zero, then decreasing, and then increasing if ϕ is relatively low but r is relatively high, and (iii) decreasing in L around zero and then increasing in L if ϕ is relatively high. It also indicates the rating becomes more informative than the no-fake benchmark as L gets large.

3.3.2 Weights on New/Previous Reviews

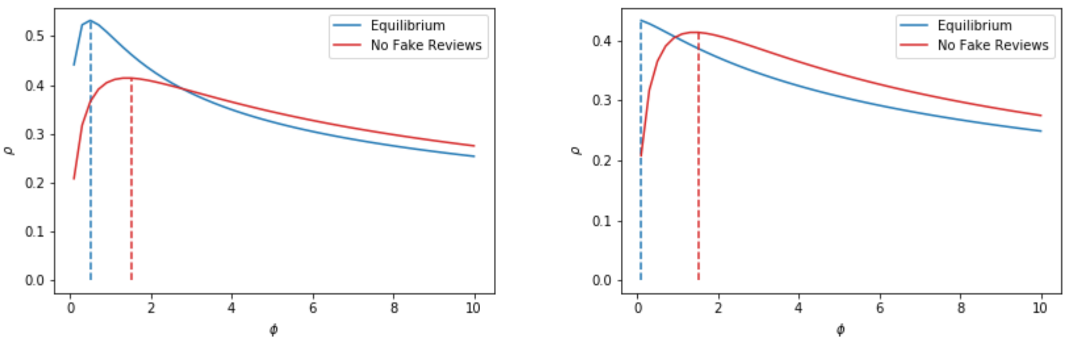
Next, we analyze the optimal weights of the new and old reviews. Again, the informativeness without fake reviews is expressed by $\rho^2\left(0;\, \phi,\, \kappa,\, \sigma_\xi,\, \sigma_\theta\right)$. Therefore, the optimal weight at this benchmark is simply characterized by $\frac{\partial}{\partial\phi}\rho^2\left(0;\, \phi,\, \kappa,\, \sigma_\xi,\, \sigma_\theta\right) = 0$. Let ϕ^0 be the solution to this equation. Meanwhile, at equilibrium, ϕ changes the equilibrium aggregator L. Thus, the optimal weight at equilibrium is characterized by $\frac{d\rho^2}{d\phi} = \frac{\partial}{\partial\phi}\rho^2\left(L;\, \phi,\, \kappa,\, \sigma_\xi,\, \sigma_\theta\right) + \frac{\partial}{\partial L}\rho^2\left(L;\, \phi,\, \kappa,\, \sigma_\xi,\, \sigma_\theta\right)\frac{dL}{d\phi} = 0$. Let the solution of this equation be ϕ^*. Now, we have the following proposition. [14]

Proposition 7. $\frac{d\rho^2}{d\phi} < 0$ *at* $\phi = \phi^0$*. Furthermore, if* r *is sufficiently small, then* $\rho^2\left(L\left(\phi^*\right);\, \phi^*,\, \kappa,\, \sigma_\xi,\, \sigma_\theta\right) > \rho^2\left(0;\, \phi^0,\, \kappa,\, \sigma_\xi,\, \sigma_\theta\right)$.

The first part of the proposition states that the platform should reduce the speed of transition ϕ, given the existence the fake reviews. Intuitively, this is explained as follows. At equilibrium, the transition of the rating score Y_t is $\phi + L$ where L is non-negative. Therefore, to cancel the strategic impact on the transition speed, the platform should decrease ϕ, compared with the no-fake benchmark ϕ^0. Again, the transition speed is interpreted as the relative weight of the new information. At the equilibrium, the number of fake reviews decreases in the current rating; thus, the fake reviews cancel the past performance. In other

[14] ϕ corresponding to disaggregated information, ϕ^d, is an alternative benchmark as in Bonatti and Cisternas (2019). In this model, we obtain a mixed result for the comparison of ϕ^* and ϕ^d. See the appendix for more details.

Figure 4: Change of the informativeness in ϕ



The left panel shows change of the informativeness in ϕ when r is relatively low, while the right panel shows that of a relatively high r. The informativeness is maximized at a lower ϕ under the equilibrium than the maximizer under the no-fake benchmark.

words, the new information is effectively weighted more than the platform intends. Thus, the platform can increase the informativeness by adjusting it downward.

The second part of the proposition is even more striking. If the seller is sufficiently concerned about the future, the platform can achieve higher informativeness than the no-fake review benchmark by adjusting the speed of updating the rating. The implication is similar to Proposition 5, but is slightly different from it. The right panel of Fig. 3 illustrates that informativeness at equilibrium is greater than that without fake reviews under some parameters (e.g., $\phi = 0.9$), as shown in Proposition 5, but it can still be lower than the maximum informativeness without fake reviews (maximized around $\phi = 1.6$). The second part of Proposition 6 states that even when we compare the maximum informativeness of the rating with and without fake reviews, the one with fake reviews will be higher if the seller cares enough about the future as shown in the left panel of Fig. 3.

3.3.3 The Precision of Genuine Reviews

Lastly, we examine the impact of the precision of organic feedback, $\frac{\sigma_\xi}{\sigma_\theta}$. As discussed in Subsection 3.2.1, increasing $\frac{\sigma_\xi}{\sigma_\theta}$ and decreasing a have similar effects on the equilibrium strategy. However, they differ in terms of the impact on informativeness. This is because a affects informativeness only through the equilibrium aggregator L, but $\frac{\sigma_\xi}{\sigma_\theta}$ affects informativeness directly as well. Intuitively, if the reviews consist of less precise feedback (i.e., higher $\frac{\sigma_\xi}{\sigma_\theta}$), the rating score, by definition, is less informative about quality. The indirect effect consists of two parts, like the comparative statics over a: (i) Higher $\frac{\sigma_\xi}{\sigma_\theta}$ implies a smaller strategic effect L, which implies less signaling effect. (ii) $L > 0$ effectively increases the rating transition to $\phi + L$. The following proposition shows that the direct effect and the first indirect effect dominate the second indirect effect for any parameter.

Proposition 8. *The informativeness at the equilibrium decreases in* $\frac{\sigma_\xi}{\sigma_\theta}$.

Thus, the precise organic feedback increases informativeness even though it comes with more fake reviews.

4 Rating Design for Naive Consumers

The model with rational consumers is a standard starting point for any economic model, but in the context of customer reviews, it is natural to consider the impact on naive consumers who do not expect any fake reviews. The regulator often tries to protect customers from fake reviews, with the assumption that the fake reviews can fool or at least confuse consumers. In this section, we assume that some consumers do not expect any fake reviews on the platform. They are modeled by assuming that the reputation (and the price) is characterized as $\widetilde{M_t} = \mu + \widetilde{\lambda}\left[Y_t - \widetilde{\nu}\right]$ where $\widetilde{\lambda}$ and $\widetilde{\nu}$ are characterized by the stochastic differential equations Eqs. (1) and (5), where $\alpha = \beta = \delta = a = 0$. Meanwhile, the long-lived seller faces the same problem as in the previous chapter, except for the definition p_t.[15]

4.1 Model / Equilibrium Characterization

In this section, the price is assumed to be a convex combination of a rational reputation M and a naive reputation $\widetilde{M}$.

$$\begin{aligned} p &= \eta M + (1-\eta)\,\widetilde{M} \\ &= \eta\left\{\mu + \lambda\left[Y_t - \nu\right]\right\} + (1-\eta)\left\{\mu + \lambda^{naive}\left[Y_t - \nu^{naive}\right]\right\} \\ &= \mu - \left(\eta\lambda\nu + (1-\eta)\,\lambda^{naive}\nu^{naive}\right) + \left(\eta\lambda + (1-\eta)\,\lambda^{naive}\right) Y_t \end{aligned}$$

One interpretation is that each consumer can be partially rational. Their expectation about the quality of the product is somewhere in between the totally sophisticated expectation and the totally naive expectation. The rationality of each consumer is captured by η.

Another interpretation is that η is the ratio of rational consumers among all consumers. Then, the market price is set somewhere in between the rational expectation and the naive expectation. When the ratio of rational consumers increases, it converges to the rational

[15] **Note to be added**: Similarity to Milgrom and Roberts (1986b) RAND “Relying on the Information of Interested Parties”]

expectation. The linear specification captures such a relationship in a simple manner. Furthermore, it can be rationalized as an equilibrium price given a specific utility function of buyers. Suppose that there are n consumers in the market and $\eta \cdot n$ of them are rational and the other $(1-\eta)\cdot n$ are naive. Consumer $i \in [0,\, n]$ feels $u_{t,i} = \theta_t + \epsilon_{t,i} - p_t$ if the consumer buys the product, and 0 otherwise, where $\epsilon_{t,i}$ is identically and independently distributed. Rational and naive consumers differ only in terms of how they form their expectation based on the same observation of the rating Y_t. Conditional on Y_t, a rational consumer purchases the product if and only if $M_t + \epsilon_i - p \geq 0$, while a naive consumer purchases it if and only if $\widetilde{M}_t + \epsilon_i - p \geq 0$. Therefore, the total demand function is expressed as

$$\eta \cdot n \cdot (1 - F(p - M)) + (1-\eta) \cdot n \cdot \left(1 - F\left(p - \widetilde{M}\right)\right)$$

where $F(p)$ is the c.d.f. of the random variable ϵ_i. By letting $n = 2q$ and assuming that ϵ_i is distributed uniformly and symmetrically around zero. We obtain $p = \eta M + (1-\eta)\widetilde{M}$ to clear the market.

In this section, we consider a linear strategy $F_t = \hat{\alpha}\theta_t + \hat{\beta} Y_t + \hat{\gamma}$ and the HJB equation with state variables θ and Y because Y keeps track of both M and $\widetilde{M}$ in a simple manner:

$$\begin{aligned} rV(\theta,\, Y) = \sup_{F\in\mathbb{R}} &(1-\tau)\, p \cdot q - \tau p \cdot F - \frac{c}{2}F^2 \\ &- \kappa(\theta - \mu) V_\theta \\ &+ \{-\phi Y_t + aF_t dt + bq\theta_t\} V_Y \\ &+ \frac{\sigma_\theta^2}{2} V_{\theta\theta} \\ &+ \frac{b^2 q^2 \sigma_\xi^2}{2} V_{YY} \end{aligned} \tag{10}$$

The following theorem states that, even with credulous consumers, we have the existence and uniqueness given the same condition as the baseline model.

Theorem 2. *For any $\eta \in [0,1]$, a stationary linear Markov equilibrium always exists. For any equilibrium, $\alpha > 0$, $\beta \in \left(-\frac{\tau}{c},\, 0\right)$, $\lambda > 0$ and $L > 0$ hold. Furthermore, if $h'(L) < 0$ holds, then such an equilibrium is unique and the equilibrium coefficients α, β, and δ are differentiable in the parameters.*

$h'(L) < 0$ holds for any $L > 0$ if $6\kappa\phi + 4r^2 + 2\kappa r + 17r\phi + 19\phi^2 > \kappa^2$.

In addition, surprisingly, the existence of naive consumers reduces the seller's strategic behavior.

Proposition 9. *The equilibrium with naive consumers ($\eta \in [0, 1)$) generates a smaller $|\alpha|$, a larger $|\beta|$,and a smaller $E[F_t]$ compared with the equilibrium without naive consumers ($\eta = 1$).*

This is because rational consumers are more sensitive to the change in ratings compared with naive consumers. Rational consumers know that the rating is boosted, but they also know that the rating is boosted more by a firm with a high quality product. Therefore, rational consumers attribute the boosted rating to high quality, and set a high price for such a boosted rating. Meanwhile, naive consumers are unaware of such a strategic correlation between quality and a rating. Therefore, with naive consumers, the price is less responsive to the boost of the ratings; thus, the seller faces a smaller *marginal* benefit of fake reviews, which leads fewer fake reviews in expectation.

Readers might wonder why the seller does not become more exploitative of naive consumers. This is simply because the fake review strategy against rational consumers generates more fake reviews for different reasons than exploiting consumers. If only a small number of naive consumers exist and observe the ratings, naive consumers would form even more biased estimates because the seller makes more fake reviews to send a good signal to rational consumers.

4.2 Optimal Rating System for Naive Consumers

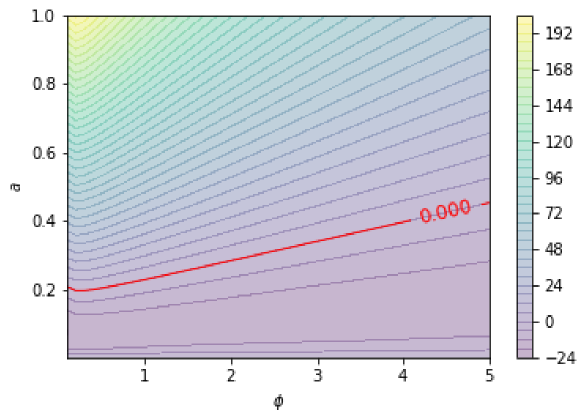
Criteria: Bias in the Reputation. In this section, we evaluate the impact of fake reviews on naive consumers. To do so, we introduce a bias in the naive consumer's expectation caused by the boosted rating:

$$\begin{aligned} Bias &\equiv E\left[\widetilde{M}_t - \theta_t\right] \\ &= E\left[\mu - \theta_t + \widetilde{\lambda}\left[Y_t - \widetilde{\nu}\right]\right] \\ &= \widetilde{\lambda}\left[\nu - \widetilde{\nu}\right] \end{aligned}$$

where $\widetilde{\lambda}$ is the sensitivity of the reputation to the rating, and ν and $\widetilde{\nu}$ are the actual mean of the rating and the estimate of the mean of the rating by the naive consumers, respectively. The decomposition of the bias, as shown above, is intuitive: the positive bias is due to the boosted reputation. Because naive consumers do not expect any fake reviews, they interpret a high rating ($Y_t > \widetilde{\nu}$) as a result of high quality, even though it is actually the average level of the rating at equilibrium ($Y_t = \nu > \widetilde{\nu}$).

Therefore, as long as the seller makes a positive amount of fake reviews (in expectation) to boost the rating, naive consumers are positively biased. This intuition is verified in the

Figure 5: Impact of censorship intensity and the weights of reviews on naive consumer's bias.



following lemma.

Lemma 5. *Bias* ≥ 0 *if and only if* $E[F_t] \geq 0$.

4.2.1 Filtering/Censoring Reviews

In the following section, for the sake of tractability, I focus on the case of $\eta = 0$, where only naive consumers exist in the market. Numerical exercises for $\eta \in (0, 1)$ can be found in the Appendix.

First, we examine the impact of a filtering policy, for which regulators are arguably concerned the most. The following proposition provides a theoretical background of a stringent policy that procect the naive customers. Note that even though the statement seems pretty intuitive, it is not trivial because the model predicts a non-monotonouse relationship between censorship and bias in general. Fortunately, in a realistic range of parameters, where naive consumers suffer from a positive bias in their reputation, stringent censorship will reduce such a bias.

Proposition 10. *Suppose Bias* ≥ 0*; then, Bias increases in* a.

Combined with Lemma 5, the condition for a stringent policy to work for naive consumers is translated as the condition of a measure observable by the platform.

Corollary 1. *Stringent censorship reduces the bias of naive consumers whenever the expected amount of fake reviews is positive.*

Thus, as long as a positive number of fake reviews are observed, the stringent policy is beneficial for naive consumers, even though it can reduce informativeness of rating for the rational consumers.

4.2.2 Weights on New/Previous Reviews

As shown in Fig. 5, the bias tends to be hump-shaped in ϕ. This is intuitive because fake reviews would be effective only when the rating is believed to be informative by the consumers so that the consumers react to the rating. Since the informativeness is hump shaped in ϕ, so is the bias caused by the fake reviews. This emphasizes that the trade-off between bias and informativeness can be an inherent feature of fake reviews.

Some readers might want an integrated criteria for bias and the informativeness. The mean squared error (MSE) is a natural candidate. It does not provide a clear-cut prediction, but a simulation of MSE is provided in the Appendix.

5 Conclusions

In this study, the effects of fake reviews on rational and credulous consumers are analyzed. The key assumption is that a high reputation results in a high cost of fake reviews. This is rationalized by the high reimbursement to reviewers or high demand for the product and the substantial, authentic feedback crowding-out the fake reviews.

At equilibrium, the amount of fake reviews increases (decreases) as product quality (firm reputation) increases (improves), which implies difficulties in the empirical analysis of signaling promotion. Stringent censorship reduces the expected amount of fake reviews, while decreasing the signaling effect and increasing the effective transition speed of the rating.

This leads to a normative result wherein the rating under a less strict filtering policy can be more informative than the rating under a strict policy or the rating with no fake reviews. In terms of the weights of new and old information in a rating system where fake reviews exist, the platform should reduce the weight of new information to maximize the informativeness of the rating, compared with a rating system that does not have fake reviews. Since fake reviews effectively attenuate the impact of old information and increase the relative weight of the new information, the platform should make the necessary adjustments.

The existence of credulous consumers decreases the expected amount of fake reviews since they are less responsive to the rating without being aware of the positive relationship between fake reviews and the quality. Moreover, they are vulnerable to fake reviews and pay more than the true quality in expectation. The model predicts that as long as a positive amount of the fake reviews is observed, the regulator or the platform can reduce such biased behaviors by enhancing censorship.

The results emphasize that regulators or platforms would face a trade-off between the degree of informativeness and the bias caused by fake reviews. As long as the rating is considered informative, the incentive to make fake reviews arises.

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A Proofs

Proof of Theorem 1. By $M_t = \mu + \lambda \left[Y_t - \nu \right] \Leftrightarrow \lambda Y_t = M_t - \mu + \lambda \nu$, and the linear strategy $F_t = \alpha \theta_t + \beta M_t + \delta \mu$, the increment of M_t is written as

$$\begin{aligned} dM_t &= d\left(\lambda Y_t\right) \\ &= \left(-\phi + a\lambda\beta\right) M_t dt \\ &+ \left(a\lambda\alpha + bq\lambda\right) \theta_t dt \\ &+ \left(\phi\mu - \phi\lambda\nu + a\lambda\delta\mu\right) dt \\ &+ bq\lambda\sigma_\xi dZ_t^\xi \end{aligned}$$

Now, we look for a quadratic value function

$$V = v_0 + v_1\theta + v_2 M + v_3\theta^2 + v_4 M^2 + v_5\theta M \tag{11}$$

satisfying the HJB equation:

$$\begin{aligned} rV\left(\theta,\, M\right) = &\sup_{F\in\mathbb{R}} \left(1-\tau\right) M\cdot q - \tau M \cdot F - \frac{c}{2}F^2 \\ &- \kappa\left(\theta - \mu\right) V_\theta \\ &+ \left\{a\lambda F + bq\lambda\theta - \phi\left[M - \bar{\theta} + \lambda\bar{Y}\right]\right\} V_M \\ &+ \frac{\sigma_\theta^2}{2} V_{\theta\theta} \\ &+ \frac{bq\lambda^2\sigma_\xi^2}{2} V_{MM} \end{aligned}$$

By the first-order condition,

$$\begin{aligned} 0 &= -\tau M - cF + a\lambda V_M \\ \Leftrightarrow F &= -\frac{\tau}{c}M + \frac{a\lambda}{c}V_M \\ &= \frac{a\lambda}{c}v_5\theta + \left(2\frac{a\lambda}{c}v_4 - \frac{\tau}{c}\right) M + \frac{a\lambda}{c}v_2 \end{aligned}$$

By matching coefficients with $F = \alpha\theta + \beta M + \delta\mu$,

$$\begin{aligned}
\alpha &= \frac{a\lambda}{c} v_5 \\
\beta &= 2\frac{a\lambda}{c} v_4 - \frac{\tau}{c} \\
\delta\mu &= \frac{a\lambda}{c} v_2
\end{aligned}$$

By solving them for v_k's,

$$\frac{c}{a\lambda}\alpha = v_5 \tag{12}$$

$$\frac{c}{2a\lambda}\left(\beta + \frac{\tau}{c}\right) = v_4 \tag{13}$$

$$\frac{\delta\mu c}{a\lambda} = v_2 \tag{14}$$

By the Envelop condition w.r.t. M,[16]

$$\begin{aligned}
rV_M = &\left(1-\tau\right) q - \tau F \\
&- \kappa\left(\theta - \mu\right) V_{\theta M} \\
&- \phi V_M \\
&+ \left\{a\lambda F + bq\lambda\theta - \phi\left[M - \mu + \lambda\nu\right]\right\} V_{MM}
\end{aligned}$$

By inserting the derivatives of eq.(11) and equating the coefficients of θ, M, and constants on LHS and RHS,

$$\begin{aligned}
\left(r+\phi\right) v_5 &= -\tau\alpha - \kappa v_5 + \left\{a\lambda\alpha + bq\lambda\right\} 2v_4 \\
2\left(r+\phi\right) v_4 &= -\tau\beta + \left\{a\lambda\beta - \phi\right\} 2v_4 \\
\left(r+\phi\right) v_2 &= \left(1-\tau\right) q - \tau\delta\bar{\theta} + \kappa\mu v_5 + \left\{a\lambda\delta\mu + \phi\mu - \phi\lambda\nu\right\} 2v_4
\end{aligned}$$

[16]The envelop condition w.r.t. θ gives conditions characterizing v_1 and v_3, and one characterizing v_5, which coincides with the condition from the envelop condition w.r.t. M.

Then, inserting eq.(12) to eq (14),

$$\left(r+\phi+\kappa\right)\frac{c}{a\lambda}\alpha=-\tau\alpha+\left\{a\lambda\alpha+bq\lambda\right\}2\frac{c}{2a\lambda}\left(\beta+\frac{\tau}{c}\right) \tag{15}$$

$$2\left(r+\phi\right)\frac{c}{2a\lambda}\left(\beta+\frac{\tau}{c}\right)=-\tau\beta+\left\{a\lambda\beta-\phi\right\}2\frac{c}{2a\lambda}\left(\beta+\frac{\tau}{c}\right) \tag{16}$$

$$\left(r+\phi\right)\frac{\delta\mu c}{a\lambda}=\left(1-\tau\right)q-\tau\delta\mu+\kappa\mu\frac{c}{a\lambda}\alpha+\left\{a\lambda\delta\mu+\phi\mu-\phi\lambda\nu\right\}2\frac{c}{2a\lambda}\left(\beta+\frac{\tau}{c}\right) \tag{17}$$

By combining with the consistency of λ: $\lambda=\frac{(a\alpha+bq)\sigma_\theta^2(\phi-a\beta\lambda)}{(\phi-a\beta\lambda+\kappa)\kappa bq\sigma_\xi^2+\sigma_\theta^2(a\alpha+bq)^2}$, we can characterize α, β, δ, λ. In the following, I do so by using an aggregator $L=-a\beta\lambda$ so that the stationarity condition is easier to verify. First, by replacing λ to $-\frac{L}{a\beta}$ in the above four equations,

$$0=-\frac{bq(\beta c+\tau)}{a}+\alpha\tau-\alpha(\beta c+\tau)-\frac{\alpha\beta c\kappa}{L}-\frac{\alpha\beta c\phi}{L}-\frac{\alpha\beta cr}{L} \tag{18}$$

$$0=\beta\tau-\beta(\beta c+\tau)-\frac{2\beta\phi(\beta c+\tau)}{L}-\frac{\beta r(\beta c+\tau)}{L} \tag{19}$$

$$0=\frac{\nu\phi(\beta c+\tau)}{a}-\delta\mu(\beta c+\tau)+\frac{\alpha\beta c\kappa\mu}{L}-\frac{\beta c\delta\mu\phi}{L}+\frac{\beta\mu\phi(\beta c+\tau)}{L}-\frac{\beta c\delta\mu r}{L}+\delta\mu\tau+q\tau-q \tag{20}$$

$$-\frac{L}{a\beta}=\frac{\sigma_\theta^2(L+\phi)(a\alpha+bq)}{\sigma_\theta^2(a\alpha+bq)^2+\kappa bq\sigma_\xi^2(\kappa+L+\phi)} \tag{21}$$

By solving (19) for β, we get $\beta=-\frac{\tau}{c}\left(\frac{r+2\phi}{r+2\phi+L}\right)\equiv B\left(L\right)$. By inserting this into (18) and solving it for α, we get $\alpha=\frac{bq}{a}\frac{L^2}{(r+2\phi)(r+\phi+\kappa+L)}\equiv A\left(L\right)$. By plugging $\beta=B\left(L\right)$ and $\alpha=A\left(L\right)$ into (21), we obtain an equation characterizing L:

$$-\frac{L}{aB\left(L\right)}=\frac{\sigma_\theta^2(L+\phi)(aA\left(L\right)+bq)}{\sigma_\theta^2(aA\left(L\right)+bq)^2+\kappa bq\sigma_\xi^2(\kappa+L+\phi)}$$

Rearranging it , we get

$$\begin{aligned}1&=\frac{\sigma_\theta^2(L+\phi)(aA\left(L\right)+bq)}{\sigma_\theta^2(aA\left(L\right)+bq)^2+\kappa bq\sigma_\xi^2(\kappa+L+\phi)}\frac{-aB\left(L\right)}{L}\\&\equiv h\left(L\right)\end{aligned}$$

To evaluate $h\left(L\right)$, the sign of L is useful to characterize.

Lemma 6. *$\beta<0$ and $L>0$ under the linear stationary Gaussian equilibrium.*

Proof. By the stationarity, we must have $\phi + L > 0$. Then,

$$\begin{aligned}\beta &= -\frac{\tau}{c}\left(\frac{r+2\phi}{r+2\phi+L}\right)\\ &= -\frac{\tau}{c}\left(\frac{r+2\phi}{r+\phi+\phi+L}\right)\\ &< 0\end{aligned}$$

Then, $\alpha = \frac{bq}{a}\frac{L^2}{(r+2\phi)(r+\phi+\kappa+L)} > 0$ and $\lambda = \frac{(a\alpha+bq)\sigma_\theta^2(\phi+L)}{(\phi+L+\kappa)b^2q^2\kappa\sigma_\xi^2+\sigma_\theta^2(a\alpha+bq)^2} > 0$. Now, we can conclude $-a\beta\lambda \equiv L > 0$. □

Now, it is shown that $\lim_{L\to 0} h(L) = \infty$ and $\lim_{L\to\infty} h(L) = 0$. Then, combined with the continuity of $h(L)$, there exist some L such that $h(L) = 1$. The uniqueness is proved by checking whether $h'(L) < 0$ holds. It is shown that

$$h'(L) = -\,h_1(L)\left\{h_2(L) + L^4\left(-\kappa^2 + 6\kappa\phi + 4r^2 + 2\kappa r + 17r\phi + 19\phi^2\right)\right\}$$

where $h_1(L),\ h_2(L) > 0$ for all $L > 0$. Thus, $6\kappa\phi + 4r^2 + 2\kappa r + 17r\phi + 19\phi^2 > \kappa^2$ is sufficient for $h'(L) < 0$ □

Proof of Lemma 2. By plugging $\alpha(L)$ and $\beta(L)$ in to h, it can be written as $h(L) = \frac{a\tau}{c}\frac{h_3}{L(L+r+2\phi)\left(h_4+\left(\sigma_\xi/\sigma_\theta\right)^2 h_5\right)}$ □

where $h_3 = (L+\phi)(r+2\phi)^2(\kappa+L+r+\phi)\left(L^2 + L(r+2\phi) + (r+2\phi)(\kappa+r+\phi)\right)$, $h_4 = bq\left(L^2 + L(r+2\phi) + (r+2\phi)(\kappa+r+\phi)\right)^2$, $h_5 = \kappa(r+2\phi)^2(\kappa+L+\phi)(\kappa+L+r+\phi)^2$. Note that h_3, h_4, h_5 are positive and independent of a and σ_ξ/σ_θ. Thus, h is increasing in $\frac{a\tau}{c}$ and decreasing in σ_ξ/σ_θ. Since $h'(L) < 0$ is shown in the proof of Theorem 1, the implicit function theorem tells that L is increasing in a and decreasing in σ_ξ/σ_θ. Furthermore, $h(L) \to \infty$ if L is bounded above and $\frac{a\tau}{c} \to \infty$. Thus, to satisfy the equilibrium condition: $1 = h(L)$, L goes infinite as $\frac{a\tau}{c}$ goes infinite. Similarly, $h(L) \to 0$ if L is bounded away from zero and $\frac{a\tau}{c} \to 0$. Thus, L goes infinite as $\frac{a\tau}{c}$ goes infinite to satisfy the equilibrium condition.

Proof of Proposition 1 and 2. Since $E[M_t] = E[E[\theta_t|Y_t]] = \mu$, we have $E[F_t] = E[\alpha\theta_t + \beta M_t + \delta\mu] = (\alpha+\beta+\delta)\mu$. By expressing α, β, δ as a function of the equilibrium aggregator L, it is written as $E[F_t] = \frac{cLq(1-\tau)(L+r+2\phi)-\mu\tau^2\left(r^2+3r\phi+2\phi^2\right)}{c\tau(L^2+L(r+2\phi)+r^2+3r\phi+2\phi^2)}$ and the partial derivative with respect to L is $\frac{\partial E[F_t]}{\partial L} = \frac{\left(r^2+3r\phi+2\phi^2\right)(2L+r+2\phi)\left(cq(1-\tau)+\mu\tau^2\right)}{c\tau(L^2+L(r+2\phi)+r^2+3r\phi+2\phi^2)^2} > 0$.

Since a, σ_ξ, and σ_θ affects $E[F_t]$ only through the aggregator L, we can show the effects of a and $\frac{\sigma_\xi}{\sigma_\theta}$ by analyzing the sign of $\frac{dL}{da}$ and $\frac{dL}{d(\sigma_\xi/\sigma_\theta)}$. By Lemma 2, we can conclude $E[F_t]$ increasing in a and decreasing in $\frac{\sigma_\xi}{\sigma_\theta}$.

Since $E[F_t] > 0$ for sufficiently large L and $L \to \infty$ as $a \to \infty$, $E[F_t] > 0$ holds for sufficiently large a. □

Proof of Proposition 3. The equilibrium condition gives $\alpha = \frac{bq}{a}\frac{L^2}{(r+2\phi)(r+\phi+\kappa+L)}$ and $\beta = -\frac{\tau}{c}\left(\frac{r+2\phi}{r+2\phi+L}\right)$. Furthermore, it is shown that $\frac{\partial\alpha}{\partial L} > 0$ and $\frac{\partial\beta}{\partial L} > 0$. Then, Lemma 2 concludes the proposition. □

Proof of Lemma 3 and 4. An arbitrary strategy α, β, δ satisfying $\phi - a\beta\lambda$ (not necessarily the equilibrium strategy) generates a stationary distribution. Using the vairance-covariance matrix of the stationary distribution, the informativeness is written as

$$\rho^2 = \frac{(\phi - a\beta\lambda)(a\alpha + bq)^2}{(\kappa + \phi - a\beta\lambda)\left((a\alpha + bq)^2 + \kappa bq\left(\sigma_\xi/\sigma_\theta\right)^2(\kappa + \phi - a\beta\lambda)\right)}$$

Thus, the informativeness without fake reviews is

$$\rho^2 = \frac{\phi(bq)^2}{(\kappa + \phi)\left((bq)^2 + \kappa bq\left(\sigma_\xi/\sigma_\theta\right)^2(\kappa + \phi)\right)}$$

.On the other hand, at the equilibrium, $-a\beta\lambda$ can be replaced to L, and $a\alpha$ is written as a function in L: $a\alpha = bq\frac{L^2}{(r+2\phi)(r+\phi+\kappa+L)}$ such that $a\alpha = 0$ when $L = 0$. Note that a does not appear in the RHS, so the direct and indirect effects of a on $a \cdot \alpha$ are all captured by L. Now the equilibrium informativeness is written as:

$$\rho^2(L; \phi, \kappa, \sigma_\xi, \sigma_\theta) = \frac{(\phi + L)(a\alpha + bq)^2}{(\kappa + \phi + L)\left((a\alpha + bq)^2 + \kappa bq\left(\sigma_\xi/\sigma_\theta\right)^2(\kappa + \phi + L)\right)}.$$

Note that $\rho^2(0; \phi, \kappa, \sigma_\xi, \sigma_\theta) = \frac{\phi(bq)^2}{(\kappa+\phi)\left((bq)^2+\kappa bq\left(\sigma_\xi/\sigma_\theta\right)^2(\kappa+\phi)\right)}$ coincides with the informativeness without fake reviews. This concludes Lemma 4. □

Proof of Proposition 5. The first part is proved by the limit as $L \to \infty$:

$$\begin{aligned}
&\lim_{L\to\infty} \rho^2 (L; \phi, \kappa, \sigma_\xi, \sigma_\theta) \\
=&\lim_{L\to\infty} \frac{(\phi + L)}{(\kappa + \phi + L)} \frac{(a\alpha + bq)^2}{\left((a\alpha + bq)^2 + \kappa bq\left(\sigma_\xi/\sigma_\theta\right)^2(\kappa + \phi + L)\right)} \\
=&1
\end{aligned}$$

The second part comes from the derivative of ρ^2 with respect to L around zero. $\square$

Proof of Proposition 6. The optimal ϕ without fake reviews is characterized by $\frac{\partial}{\partial\phi}\rho^2 (0; \phi, \kappa, \sigma_\xi, \sigma_\theta) = 0$, which yields $\phi^0 = \sqrt{bq\left(\sigma_\theta/\sigma_\xi\right)^2 + \kappa^2}$ as the optimal level. On the other hand, the effect of ϕ at the equilibrium is

$$\begin{aligned}
\frac{d\rho^2}{d\phi} &= \frac{\partial}{\partial\phi}\rho^2 (L; \phi, \kappa, \sigma_\xi, \sigma_\theta) + \frac{\partial}{\partial L}\rho^2 (L; \phi, \kappa, \sigma_\xi, \sigma_\theta) \frac{dL}{d\phi} \\
&= \frac{\partial}{\partial\phi}\rho^2 (L; \phi, \kappa, \sigma_\xi, \sigma_\theta) - \frac{\partial}{\partial L}\rho^2 (L; \phi, \kappa, \sigma_\xi, \sigma_\theta) \frac{\partial h}{\partial\phi} \Big/ \frac{\partial h}{\partial L}
\end{aligned}$$

By evaluating this at $\phi = \phi^0$, we obtain $\frac{d\rho^2}{d\phi}|_{\phi=\phi^0} < 0$.

The second part is proved by two inequalities: $\rho^2 (0; \phi^0, \kappa, \sigma_\xi, \sigma_\theta) < \rho^2 (L(\phi^0); \phi^0, \kappa, \sigma_\xi, \sigma_\theta) \le \rho^2 (L(\phi^*); \phi^*, \kappa, \sigma_\xi, \sigma_\theta)$. The first inequality is proved as follows. For any $L > 0$,

$$\begin{aligned}
&\rho^2 \left(L; \phi^0, \kappa, \sigma_\xi, \sigma_\theta\right) - \rho^2 \left(0; \phi^0, \kappa, \sigma_\xi, \sigma_\theta\right) \\
=&r \cdot g_1 + g_2
\end{aligned}$$

where g_1 is polynomial in r and L and $g_2 > 0$ is polynomial in L and does not depend on r. Since $L \to C$ for some $C > 0$ as $r \to 0$, $r \cdot g_1 + g_2$ converges to a positive number. Thus, for sufficiently small r, the first inequality holds. The second inequality holds by definition. $\square$

Proof of Proposition 7. Similarly to Proposition 6, the total effect of σ_ξ/σ_θ is written as $\frac{d\rho^2}{d(\sigma_\xi/\sigma_\theta)} = \frac{\partial}{\partial(\sigma_\xi/\sigma_\theta)}\rho^2 (L; \phi, \kappa, \sigma_\xi, \sigma_\theta) - \frac{\partial}{\partial L}\rho^2 (L; \phi, \kappa, \sigma_\xi, \sigma_\theta) \frac{\partial h}{\partial(\sigma_\xi/\sigma_\theta)} / \frac{\partial h}{\partial L}$. It is shown that $\frac{d\rho^2}{d(\sigma_\xi/\sigma_\theta)} < 0$. $\square$

Proof of Theorem 2. Now, we look for a quadratic value function

$$V = v_0 + v_1\theta + v_2 Y + v_3\theta^2 + v_4 Y^2 + v_5\theta Y \tag{22}$$

satisfying the HJB equation:

$$\begin{aligned} rV(\theta, Y) = \sup_{F\in\mathbb{R}} & (1-\tau)\, p\cdot q - \tau p\cdot F - \frac{c}{2}F^2 \\ & - \kappa(\theta-\mu) V_\theta \\ & + (aF + bq\theta - \phi Y) V_Y \\ & + \frac{\sigma_\theta^2}{2} V_{\theta\theta} \\ & + \frac{bq\sigma_\xi^2}{2} V_{YY} \\ \text{s.t. } & p = \mu - \left(\eta\lambda + (1-\eta)\tilde{\lambda}\right) Y + \left(\eta\lambda\nu + (1-\eta)\tilde{\lambda}\tilde{\nu}\right) \end{aligned}$$

The first order condition and gives

$$v_5 = \frac{\alpha c}{a} \tag{23}$$

$$v_4 = \frac{\beta c + \hat{\lambda}\tau}{2a} \tag{24}$$

$$v_2 = \frac{c\delta\mu + \mu\tau - \widehat{\lambda\nu}\tau}{a} \tag{25}$$

where $\hat{\lambda} = \left(\eta\lambda + (1-\eta)\tilde{\lambda}\right)$ and $\widehat{\lambda\nu} = \left(\eta\lambda\nu + (1-\eta)\tilde{\lambda}\tilde{\nu}\right)$, and the envelop condition gives

$$0 = \hat{\lambda}\alpha\tau - 2a\alpha v_4 - 2bqv_4 + rv_5 + \kappa v_5 + v_5\phi \tag{26}$$

$$0 = -2a\beta v_4 + \beta\hat{\lambda}\tau + 2rv_4 + 4v_4\phi \tag{27}$$

$$0 = -2a\delta\mu v_4 + \delta\mu\hat{\lambda}\tau + \hat{\lambda}q\tau - \hat{\lambda}q + rv_2 - \kappa\mu v_5 + v_2\phi \tag{28}$$

By inserting eq.(24) into (27) and solving it for $\hat{\lambda}$ and by letting $L = a\beta$, we obtain

$$\hat{\lambda} = \frac{cL(L + r + 2\phi)}{a\tau(r + 2\phi)} \equiv \hat{\lambda}(L)$$

On the other hand, the stochastic differential equation for (θ, Y) gives

$$\lambda = \frac{bq\sigma_\theta^2(L+\phi)\left(A\left(L\right)+1\right)}{\sigma_\theta^2\left(bqA\left(L\right)+bq\right)^2+\kappa bq\sigma_\xi^2(\kappa+L+\phi)} \equiv \lambda\left(L\right)$$
$$\tilde{\lambda} = \frac{bq\sigma_\theta^2\phi}{\sigma_\theta^2\left(bq\right)^2+\kappa bq\sigma_\xi^2(\kappa+\phi)} = \lambda\left(0\right)$$

Then, by rearranging

$$\hat{\lambda} = \left(\eta\lambda+\left(1-\eta\right)\tilde{\lambda}\right)$$
$$\Rightarrow 1 = \frac{\eta\lambda\left(0\right)+\left(1-\eta\right)\lambda\left(L\right)}{\hat{\lambda}\left(L\right)} \equiv h\left(L;\eta\right)$$

Note that $\lim_{L\to 0} h\left(L;\eta\right)=\infty$ and $\lim_{L\to\infty} h\left(L;\eta\right)=0$. Then, $h_L\left(L;\eta\right)<0$ holds for any $\eta\in[0,\,1]$ as long as $h_L\left(L;1\right)<0$. □

Proof of Proposition 8. Since $\lambda\left(0\right)\leq\lambda\left(L\right)$ for any $L\geq 0$, we have $h\left(L;\eta\right)\leq h\left(L;1\right)$ for any $\eta\in[0,\,1]$. Thus, the equilibrium L will be smaller given $\eta<1$ than the equilibrium L given $\eta=1$.

The expected amount of the fake reviews is

$$E\left[F_t\right]=\alpha\mu+\beta\nu+\delta\mu$$

By plugging the equilibrium conditions and taking derivative with respect to L, we can show $\frac{\partial}{\partial L}E\left[F_t\right]\geq 0$. □

Proof of Proposition 9. At the equilibrium, $\frac{\partial bias}{\partial L}\geq 0$ always holds and $\frac{\partial bias}{\partial a}\geq 0$ holds if $bias\geq 0$. □

B An interpretation of the pricing rule

this pricing rule as a result of competition among heterogeneous consumers, to which we can easily introduce a mixture of rational and naive consumers in the next section. Suppose that consumer $i\in[0,\,n]$ feels $u_{t,i}=\theta_t+\epsilon_{t,i}-p_t$ if the consumer buy the product, and 0 otherwise, where $\epsilon_{t,i}$ is identically and independently distributed. Then, given the rating shown on the platform, Y_t, the consumer will choose to purchase the product if and only if

$E[\theta_t|Y_t] + \epsilon_{t,i} - p_t \geq 0$. Therefore, the demand function is expressed as $n \cdot (1 - F(p_t - M_t))$ where $F(\cdot)$ is a c.d.f. of the random variable $\epsilon_{t,i}$. By letting $n = 2q$ and assuming that $\epsilon_{t,i}$ is distributed symmetrically around zero. We obtain $p_t = M_t$ as the market clearing price.

C An Alternative Model with Changing q

The same results with the base line model can be generated with a slightly different specification of the model with the quantity level dependent on the reputation level.

Now, suppose that the seller sells q_t units of the product at a fixed price of p, and makes F_t units of fake reviews. The quality of the product is denoted as θ_t. A sufficiently large mass of consumers forms a belief on the quality $E[\theta_t|Y_t] \equiv M_t$ and the demand function based on that. Since the price is fixed, high reputation results in large quantity: $q_t = M_t$.

The quality θ_t evolves in the same way as the main model. The new information as

$$aF_t dt + bq_t \left(\theta_t dt + \sigma_\xi dZ_t^\xi\right) \tag{29}$$

The difference from the main model is that the quantity varies over time and the coefficient of dZ_t^ξ is now defined as $bq_t\sigma_\xi$ instead of $\sqrt{bq_t}\sigma_\xi$. In this specification, we can analyze the effect of the organic reviews crowding out the fake reviews, but not the effect of the large transaction generating intrinsically more precise information by the large sample.

The seller's instantaneous payoff is defined as:

$$\pi_t = (1-\tau)\, p\, (q_t + F_t) - p \cdot F_t - \frac{c}{2}\left(\frac{F_t}{q_t}\right)^2$$

where τ is transaction fees imposed by the platform. The specification of the quadratic cost is now different from the base line model: the seller needs to pay a large cost if the seller tries to increase the share of the fake reviews among the all the reviews. The revenue and the reimbursement cost is still the same as the baseline model.

$$\begin{aligned}\pi_t &= (1-\tau)\, pq_t - \tau p \cdot F_t - \frac{c}{2}\left(\frac{F_t}{q_t}\right)^2 \\ &= (1-\tau)\, pM_t - \tau p \cdot M_t \frac{F_t}{M_t} - \frac{c}{2}\left(\frac{F_t}{M_t}\right)^2\end{aligned}$$

By changing the choice variable of the seller from F_t to $\frac{F_t}{M_t}$, which is the combination of the original variable and a constant at time t, we can write the instantaneous profit isomorphic to one in the baseline model. To simplify the analysis, we assume that the platform use an average information at time t to update the ratings:

$$d\xi = \frac{a}{b}\frac{F_t}{M_t}dt + \theta_t dt + \sigma_\xi dZ_t^\xi \tag{30}$$

The model is then isomorphic to the baseline model, so generates the same results as those from the baseline model.

D Simulation Results

D.1 Mixture of the Rational and Naive Consumers

In the main part, the correlation of the rating with the underlying true quality for rational consumers, and the bias for the naive consumers are examined. There is a trade-off of the correlation and the bias. Then, natural questions are (i) how to integrate such indices into one objective function, and (ii) how it changes as the market's rationality changes from totally naive to totally rational. In this section, we suggest a mean squared error of the price since the price is considered as the whole market's prediction about the underlying quality. The minimization of the mean squared errors minimizes the customers' *ex post* regret on average, so increases the value-added of the platform, and attracts the customers in long-run.

D.1.1 Mean Squared Error

The mean squared errors of the price is defined and written with the equilibrium variables as follows:

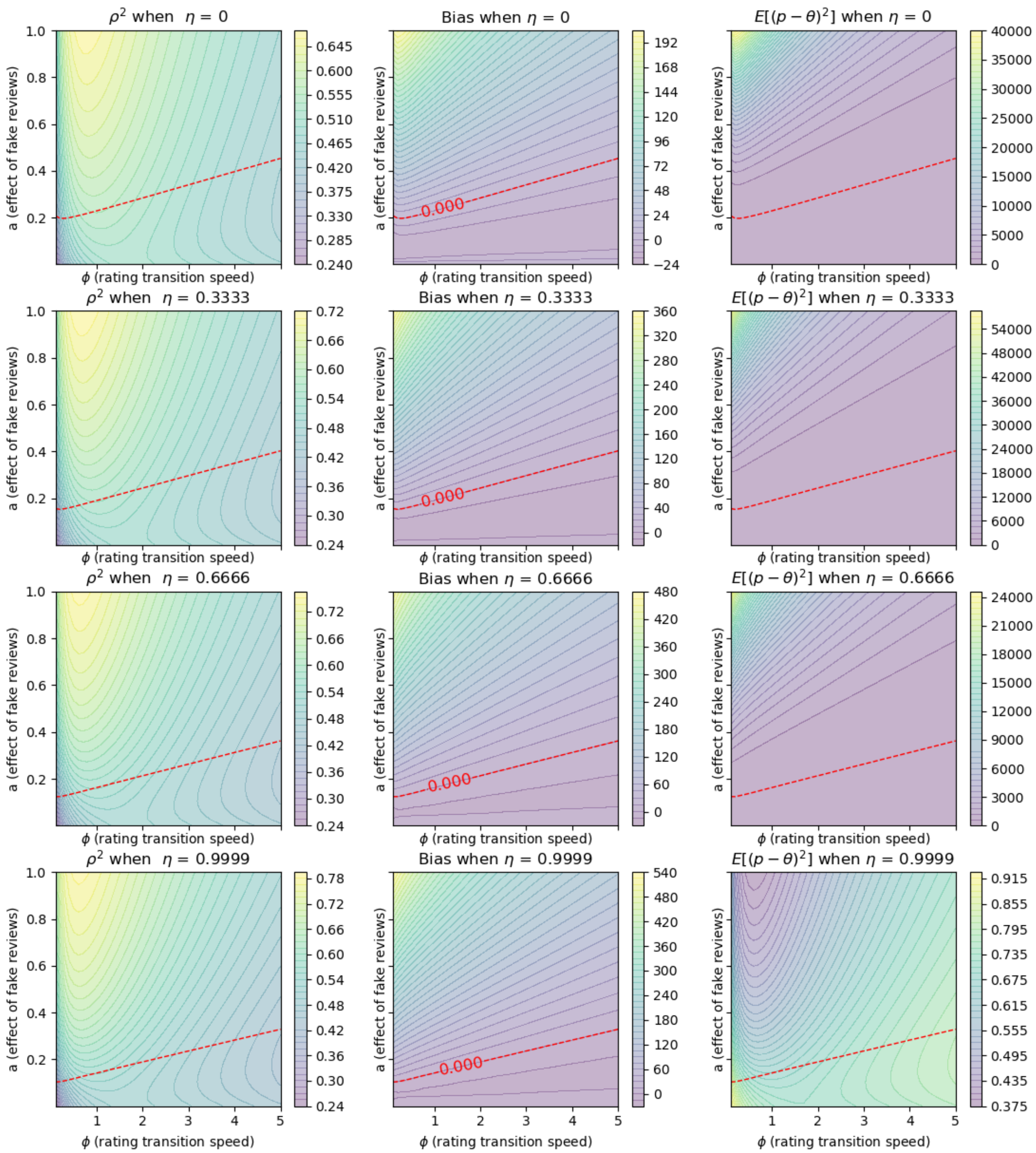
$$\begin{aligned}
MSE_p &= E\left[(p_t - \theta_t)^2\right] \\
&= E\left[\left(\eta\left\{\mu + \lambda\left[Y_t - \nu\right]\right\} + (1-\eta)\left\{\mu + \widetilde{\lambda}\left[Y_t - \widetilde{\nu}\right]\right\} - \theta_t\right)^2\right] \\
&= Var\left(Y_t\right)\left\{\left(\eta\lambda + (1-\eta)\widetilde{\lambda}\right)^2 - 2\left(\eta\lambda + (1-\eta)\widetilde{\lambda}\right)\lambda\right\} + (1-\eta)^2 Bias^2 + Var\left(\theta_t\right)
\end{aligned}$$

Note that, when $\eta = 1$, minimization of MSE is reduced to maximization of the correlation of the rating Y_t and θ_t:

$$
\begin{aligned}
MSE_p &= -\lambda^2 Var\left(Y_t\right) + Var\left(\theta_t\right) \\
&= Var\left(\theta_t\right)\left\{1 - \frac{Cov\left(Y_t, \theta_t\right)^2}{Var\left(Y_t\right)^2}\frac{Var\left(Y_t\right)}{Var\left(\theta_t\right)}\right\} \\
&= Var\left(\theta_t\right)\left\{1 - \rho^2\right\}
\end{aligned}
$$

For different levels of η, we calculate the correlation of Y and θ as a criteria for the rational consumers, the bias as a criteria for naive consumers, and the mean squared error as a criteria for the whole market. See fig.5 for the simulation results. The correlation of the rating with the underlying quality show the similar pattern regardless of the level of η, while it is scaled up as the rationality increases. So does the bias the naive consumers faces. This is consistent with Proposition 9. As the market becomes more rational, the consumers takes the signaling effect of the seller's fake reviews ($\alpha > 0$), so the market becomes more sensitive to the rating. Then, the seller will have more incentive to make fake reviews, resulting in more bias for naive consumers. At the same time, the signaling effect ($\alpha > 0$) is also enhanced by this increased manipulation by the seller. Therefore, the rating becomes more informative for rational consumers. Roughly speaking, the mean squared error integrates the correlation and the bias into one. As the ratio of the rational consumers increases, the correlation becomes more important. As the ratio of the naive consumers increases, the bias comes more important. Fig. 5 exhibits this. For $\eta = 0$, 0.3333, 0.6666, the MSE shows the similar pattern as the bias, while the MSE shows the similar pattern as the correlation for $\eta = 0.9999$. Given other parameters used in the simulation, the bias is the dominant force in MSE for most of η. This results depends on the parameter setting, so is ultimately an empirical question, but suggests that decreasing the bias is more important than increasing the informativeness for rational consumers.

Figure 6: Correlation, bias, and mean squared errors



From top to the bottom, the rationality of the market is increased from 0, 0.3333, 0.6666, to 0.9999. The left panels are contours of the correlation of the rating Y_t with θ_t based on rational expectations taking the seller's strategy into account. The middle panels show biases the naive consumers faces. The right panels show the mean squared errors of the market price as a whole market's prediction of the underlying quality. Red dashed lines border sets of parameters which predict realistic positive bias (positive number of positive fake reviews) at the equilibrium. Areas above red lines corresponds to the positive number of positive fake reviews.

Optimal tariff policies with emission taxes under non-restrictive two-part licensing strategies by a foreign eco-competitor

Seung-Leul Kim* and Sang-Ho Lee†

Abstract

This paper investigates eco-technology licensing strategy by a foreign innovator that constructs two-part licensing contract when it competes with a polluting domestic firm in the home country. In particular, we consider and compare the two-part licensing contracts with and without non-negative constraints, and find that the foreign eco-innovator strategically might choose either negative royalty or negative fixed fee, depending on the levels of emission tax and tariff. We then examine the government's optimal trade policies under the emission tax and show that non-restrictive two-part licensing contract is better off to the domestic welfare than that with restrictive licensing contract. Finally, we show that the optimal tariff policy under the two-part licensing has a negative relation with emission taxes, but the tariff under the non-restrictive licensing is a higher than that under the restrictive licensing.

JEL Classification: L13; D45; H23

Keywords: Eco-technology; tariff policies; emission tax; non-restrictive two-part licensing; foreign innovated firm

1. Introduction

During the last generation, technology innovation and free trade stance have drastically expanded the volume of international trade and globalization all over the world. Owing to the liberalization and deregulation of economic activities, however, the scope and nature of trade and environmental problems have also been diversified without being limited to a specific region or country.

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As the environmental problems such as climate change are restricted by greenhouse gas (GHG) reduction plans and other environmental regulations in the world. The GHG reduction plans are primarily focused on industrial R&D activities aimed at reducing emission levels. Many countries and companies are eager to develop eco-friendly technologies and products to reach near-zero emissions. International organizations and research institutions are also emphasizing the importance of the eco-technology industry to reduce global pollution emissions.

Meanwhile, many countries are strengthening the introduction of technology barriers to trade (TBT) as one of their trade protection policies. The industrial regulations and policies on environmental technology have become an international issue. In the rapidly changing global economy, the combined policy of trade and environmental issues caused by such globalization is one of the most important economic growth policies.

The interaction between trade policy and environmental regulation has been studied in the academic literature. The early studies focused on the interest of government revenue in how to adjust tariffs and emissions taxes to improve welfare. (see Copeland, 1994; Gulati and Roy, 2008; Hatzipanayotou, 2009). Recent works examined the optimal policies and showed that trade liberalization results in less-stringent environmental regulations, which suggests that policies regulating trade and the environment are positively correlated. Tsai et al. (2014), however, showed that positive relationship between tariffs and environmental taxes may not be applicable under the eco-technology which can fully abate pollution through ER&D. When both environmental taxes and tariffs are employed, Chao et al. (2012) also showed that welfare is maximized with the first-best optimal policy which is free trade and a Pigouvian tax on consumption-based pollution.

One the other hand, one of the most important global phenomena is the significant increase in the volume and value of patent licensing in recent years. (see Zuniga and Guellec, 2009). Patent licensing is an important policy issue for improving social welfare as well as protecting the interests of innovators because patents can provide incentives for effective R&D to develop new products or technological innovations. For example, Kabiraj and Marjit (2003) and Mukherjee and Tsai (2013) examined the effect of tariff or subsidy/tax policies on technology licensing decision and showed that innovators license to domestic licensees in the presence of optimal policies. Kabiraj and Kabiraj,(2017) also showed that the government can impose a positive tariff on importing goods under the two-part licensing contracts, which can induce fixed fee licensing of the foreign licensor and maximize domestic welfare.

This study intends to research eco-technology licensing under trade and environmental policy and examines the relationship between tariffs and emission taxes under the licensing strategies of eco-technology. In particular, we investigate strategic two-part licensing contracts by a foreign innovator, which competes with a polluting domestic firm in the home country. We also examine policy relations between tariffs and emission taxes facing when the government coordinate the optimal policy with the introduction of eco-technology. Intuitively, a licensing contract increases the production cost of domestic firms while a tariff policy raises the cost of foreign firms. Also, as the increase in emission tax, domestic firms decrease to produce output, but foreign firms produce more outputs and receive more licensing fee. As a result, social welfare depends not only on the effect of reducing pollution damage but on the effect of extracting profit to the foreign firm.

In particular, in the presence of environmental regulation and trade policy, we examine two-part licensing contracts by a foreign eco-competitor and compare both restrictive contract with non-negative constraint and non-restrictive contract without non-negative constraint. [1] We then find how the polluting domestic firm may purchase a license of eco-technology from the foreign firm with response to tariff and emission tax. We show that the foreign eco-innovator strategically might choose either negative royalty or negative fixed fee, depending on the levels of emission tax and tariff. We then examine the government's optimal trade policies under the emission tax and show that non-restrictive two-part licensing contract is better off to the domestic welfare than that with restrictive licensing contract. We also show that the optimal tariff policy under the two-part licensing has a negative relation with emission taxes, but the tariff under the non-restrictive licensing is a higher than that under the restrictive licensing. Finally, we show that the market equilibrium might yield welfare loss for some ranges of trade policy and environmental regulation under the both restrictive and non-restrictive contracts.

The remainder of this paper is organized as follows. In section 2, we provide a literature review. We provide the basic model in section 3. We examine no licensing case in section 4 while construct the two-part licensing contracts with a restrictive case in section 5 and with

[1] Kabiraj and Kabiraj (2017) and Yang *et al*. (2020) constrained their analysis into the case of non-negative royalty and fixed fee, while Lia and Sen (2005) and Hattori and Tanaka (2018) considered negative royalty and showed that a subsidized royalty can be an equilibrium strategy of the inside innovator.

non-restrictive case in section 6, respectively. In section 7, we compare the equilibrium outcomes under both the restrictive and the non-restrictive two-part licensing contracts. In section 8, we discuss welfare consequences of the optimal two-part licensing contracts and explain the interaction between two policy instruments. The final section provides a conclusion.

2. Literature Review

The theoretical literature in patent licensing, the innovators can give a license to licensees by means of different licensing contracts such as royalty, fixed-fee licensing, auctioning, two-part tariff licensing and so on. However, with respect to empirical evidence on licensing contracts, it reveals that most of the contracts include a positive royalty and the combinations of up-front fees and royalties. For example, Rostoker (1984) surveyed that two-part tariff licensing (including royalty plus fixed fee) was used 46%, royalty alone 39%, and fixed fee alone 13% among 37 corporations in manufacturing industry. Bousquet et al. (1998) show using French data that in a sample of 278 contracts, 225 (82%) includes royalties of which 216 (96%) are ad valorem. Vishwasrao (2007) also shows that patent licensors are empirically more likely to ask for royalties when sales are relatively high and involatile but profits are low.

Early work shows that with outside innovation, fixed fee policy is superior to royalty (or auction) policy in perfect competition (Kamien and Tauman, 1984; Katz and Shapiro, 1985), homogenous oligopoly model (Kamien and Tauman,1986; Katz and Shapiro, 1986; Kamien *et al.*, 1992), asymmetric Cournot industry (Stamatopoulos and Tauman, 2009), eco-industry (Kim and Lee, 2014). But, there are still debates on these results since royalty is preferred to fixed-fee in Bertrand model (Muto, 1993), product differentiation model (Poddar and Sinha, 2004, Bagchi and Mukherjee, 2010), open economy (Mukherjee, 2007), and dynamic frameworks (Saracho, 2011). Others have also focused on patent licensing about internal innovation. They show that royalty licensing is preferred to fixed-fee licensing in a Cournot duopoly with a homogeneous good (Wang, 1998), a differentiated Bertrand duopoly (Wang & Yang, 1999), incumbent innovators in a homogeneous Cournot oligopoly (Kamien & Tauman, 2002), and a leadership structure (Kabiraj, 2005). However, Wang (2002) shows that fixed-fee licensing is preferred to royalty licensing for an internal patentee with a heterogeneous duopoly if the product differentiation is sufficiently large.

While previous literature mainly deals with royalty and fixed-fee licensing, recent researches in industrial economics have focused on two-part licensing, basically consisting of a fixed fee plus a per-unit royalty. Erutku and Richelle (2007) also show that an outside innovator always prefers a fixed-fee plus a royalty contract, which gives profit a monopoly endowed with the innovation but can reduce social welfare. Sen and Tauman (2007) show that licensing for a cost reduction innovation under combinations of upfront fees (auctioning fee) and royalties unambiguously leads to improvement of social welfare in a homogenous oligopoly. Fauli-Oller *et al*. (2012) point out that the innovation is licensed to all firms under two-part tariff, regardless of the number of firms, the degree of product differentiation and the type of patentee. Moreover, two-part tariff licensing can be developed by ad valorem royalties with a fee (Hernandez-Murillo and Liobet, 2006; Martin and Saracho, 2015), unionized labor market (Mukherjee, 2010), leadership duopoly model with product differentiation (Li and Yanagawa, 2011), and homogeneous oligopoly market in the presence of tax and subsidy policy (Mukererjee and Tsai, 2013).

Theoretical literature has also analyzed the relationship between market structure and regulatory policy in order to compare the efficiencies of two-part licensing. With inside innovation, many studies analyze the two-part licensing in the differentiated Cournot and Bertrand duopolies (Mukherjee and Balasubramanian, 2001; Fauli-Oller and Sandonis, 2002) , in Cournot duopoly with homogenous goods (Martin and Saracho, 2010), in a differentiated Cournot duopoly with ad valorem and unit royalty (Martin and Saracho, 2015), in environmental patent technology (Kim and Lee, 2016).

There are also some studies of trade policy with licensing. Kabiraj and Marjit (2003) and Mukherjee and Pennings (2006) show that the role of government in technology licensing under an open economy. In such an economy, tariff policy induces fee licensing than royalty licensing with consideration of maximizing domestic welfare. Meanwhile, Wang et. al (2012) examine the relation of strategic trade policy and welfare with consumer-friendly initiative of foreign exporting firm. Recent work shows that with two-part licensing of cost-reducing technology, a tariff can be chosen to induce fee licensing and maximize both consumers' surplus and domestic welfare in an international duopolistic model (Kabiraj and Kabiraj, 2017), foreign Stackelberg leadership model (Yang et al., 2020). Our model is close to these works wherein the government imposes a positive tariff on importing goods under two-part licensing.

We analyze the total effect of two policy instruments which are tariff and emission tax with eco-technology. We also extend the assumption of a restrictive two-part licensing into a non-restrictive case in which either a negative royalty or a negative fixed-fee is possible.

3. Model

Consider a Cournot duopoly where a domestic firm and a foreign firm compete in a domestic market with homogenous products that might emit pollutants. The inverse demand function in the domestic market is given by $P = A - Q$, where $Q = q_d + q_f$ denotes market outputs and q_d and q_f are the outputs of the domestic and foreign firms, respectively. We assume that both firms have the same constant marginal cost, c.

While the domestic firm emits pollutants, the foreign firm is an eco-firm and has a patent of environmentally clean technology, i.e., zero-pollution eco-technology that produces no emission, for simplicity. The foreign firm can consider a technology licensing strategy where it can license its eco-technology to its rival firm by offering licensing contract of two-part scheme consisting of per-unit royalty and fixed-fee. If the domestic firm adopts the licensing contract, the licensed domestic firm can also reduce pollution (and expenditure on emission tax if the government imposes emission tax) while the non-licensed domestic firm will continue to emit pollution where its emission function is defined as $E = q_d$.

We denote environmental damage as $D(E) = dE$, which is constant to the total emission level. The social welfare function will be defined as the sum of consumer surplus, domestic firm's profit and government total revenue minus environmental damage. We assume that the government maximizes domestic welfare and imposes an emission tax at the rate of t (> 0) on domestic firm. Further, the government can impose an import tariff at the rate of τ (> 0) on foreign products. We assume that $0 \leq \tau < (a-c+t)/2$ to assure the interior solutions in the analysis.

We examine two-part licensing contracts by the foreign licensor but compare the two cases: (i) restrictive scheme with non-negative constraints on per-unit royalty and fixed-fee, and (ii) non-restrictive scheme without non-negative constraints. We then examine the optimal government policies between emission tax and tariff.

The timing of the game is as follows. In the first stage, for given emission tax and tariff, the foreign firm announces the provision of eco-technology and decides a per-unit royalty and a fixed-fee. In the second stage, given the two-part licensing contract, the domestic firm decide whether to purchase a license. Finally, each domestic and foreign firm chooses output levels q_d and q_f in a Cournot-fashion in the last stage. The sub-game perfect Nash equilibrium will be divided by backward induction.

4. No licensing case

Under no licensing, a foreign firm faces an import tariff while a domestic firm faces an emission tax in the output competition. The profit functions of a domestic firm and a foreign firm are as follows, respectively:

$$\pi_d^N = P(Q)q_d^N - cq_d^N - tq_d^N \quad \text{and} \quad \pi_f^N = P(Q)q_f^N - cq_f^N - \tau q_f^N \tag{1}$$

Then, the equilibrium quantities of the firms are as follows:

$$q_d^N = \frac{a-c-2t+\tau}{3} \quad \text{and} \quad q_f^N = \frac{a-c+t-2\tau}{3} \tag{2}$$

where N denotes the equilibrium under no licensing. It shows that the relative size between t and τ determines the production and profitability. That is, $q_d^N \overset{>}{\underset{<}{-}} q_f^N$ if $\tau \overset{>}{\underset{<}{-}} t$. For non-negative quantities of both firms at equilibrium, we will assume the followings:

$$\max\left[0\,,\,2t+c-a\right] \le \tau < \frac{a-c+t}{2}. \tag{3}$$

Let us define $\tau_D = \dfrac{a-c+t}{2}$ and $\tau_M = 2t+c-a$. Then, the domestic firm will act as a monopolist if the tariff is prohibitive, i.e., $\tau \ge \tau_D$, while the foreign firm will act as a monopolist. if the emission tax is very high, i.e., $t > \dfrac{a+\tau-c}{2}$ (or $\tau \le \max[0\,,\tau_M\,]$).

The corresponding profits of the firms and the welfare are as follows:

$$\pi_d^N = \left(\frac{a-c-2t+\tau}{3}\right)^2 \quad \text{and} \quad \pi_f^N = \left(\frac{a-c+t-2\tau}{3}\right)^2 \tag{4}$$

$$W^N = \int_0^Q P(u)du - cq_d + \tau q_f - dE$$
$$= \frac{1}{6}\left[2(a-c)^2 - 2d(a+\tau-2t-c) + 2\tau(a+t-c) - t(2a-2c+t) - 3\tau^2\right] \tag{5}$$

5. Restrictive two-part licensing case with $r \geq 0$ and $f \geq 0$

We will consider two-part licensing contract $T(r, f)$ where r is a per-unit royalty and f is a fixed-fee. Note that we impose a non-negativity constraint of the royalty and fee. The profit functions of a foreign firm and a licensed domestic firm are as follows, respectively:

$$\pi_d^T = P(Q)q_d - cq_d - rq_d - f \quad \text{and} \quad \pi_f^T = P(Q)q_f - cq_f - \tau q_f + rq_d + f \tag{6}$$

The equilibrium quantities of the firms are as follows:

$$q_d^T = \frac{a-c-2r+\tau}{3} \quad \text{and} \quad q_f^T = \frac{a-c+r-2\tau}{3} \tag{7}$$

where T denotes the equilibrium under two-part licensing. Then, it shows that the relative cost relation between r and τ determines the efficiency of the firm and profitability. That is, $q_d^T \gtreqless q_f^T$ if $\tau \gtreqless r$. It is noteworthy that the foreign firm can strategically manipulate the royalty in order to increase (decrease) the output of licensee, which yields asymmetric cost between the firms, and then impose a fixed-fee to retrieve the increased profit of the domestic firm into its profit.

The corresponding profits of the firms become as follows:

$$\pi_d^T = \left(\frac{a-c-2r+\tau}{3}\right)^2 - f \quad \text{and} \quad \pi_f^T = \left(\frac{a-c+r-2\tau}{3}\right)^2 + rq_d + f \tag{8}$$

where $r \geq 0$ and $f \geq 0$. In order to make a contract agreeable to the domestic firm, the fixed-fee should be equal to the profit difference between accepting and rejecting the licensing offer, i.e., $\pi_d^T - \pi_d^N = 0$ at equilibrium. It implies that the contract should ensure the incentive compatibility of the licensed domestic firm between licensed case and non-licensed case. Then, we have the following relations between the fixed-fee and royalty:

$$f(r)=\frac{4}{9}(r-t)(r+c+t-a-\tau) \tag{9}$$

Note that the relation between the fixed-fee and royalty is non-monotone and U-shape, i.e., as the royalty increases, the fixed-fee decreases first and then increases later, given the policy parameters.[2] Then, given this constraint in (9), the foreign firm will determine the royalty to maximize the equilibrium profit function in (8):

$$\max_{r}\left(\frac{a-c+r-2\tau}{3}\right)^2+rq_d^T+f(r) \quad \text{s.t} \quad f(r) \quad \text{in (9)} \tag{10}$$

From the first-order condition, we have the following optimal royalty:

$$r^*=r=\frac{a-c-5\tau}{2} \tag{11}$$

Note that r^* is decreasing in τ. Note also that with a non-negative constraint on r, the optimal royalty should be positive, i.e., $r\geq 0$ if $\tau\leq\frac{(a-c)}{5}$. Then, we can construct the optimal fixed-fee f^* as follows:

$$f^*=f(r)=\frac{(5\tau-a+2t+c)(a-2t-c+7\tau)}{9} \tag{12}$$

Note that the changes of f^* have the positive relation between τ and t.[3]

Let us define τ^R as a royalty level with zero fixed fee, which satisfies $f^*=0$, and τ^F as a fixed fee level with zero royalty, which satisfies $r^*=0$.

$$\tau^R=\frac{a-c-2t}{5} \quad \text{and} \quad \tau^F=\frac{a-c}{5} \tag{13}$$

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[2] It is easy to check $\frac{\partial^2 f(r)}{\partial r^2}>0$ and if $r \gtrless \frac{a+\tau-c}{2}$, then $\frac{\partial f(r)}{\partial r}\lessgtr 0$. Also, we have $\frac{\partial^2 f(r)}{\partial r\,\partial\tau}<0$.

[3] We have $\frac{\partial^2 f^*}{\partial t^2}<0$, $\frac{\partial^2 f^*}{\partial\tau^2}>0$, and $\frac{\partial^2 f^*}{\partial\tau\,\partial t}>0$.

Also note that τ^R can be positive (if $t < \frac{(a-c)}{2}$) or negative (if $t > \frac{(a-c)}{2}$) while τ^F is always positive. Then, Fig. 1 provides the relations of the findings where τ^R and τ^F are the boundaries of duopolistic competition. That is, if the tariff is very high ($\tau \geq \tau_D$), only domestic firm exists in the market while if emission tax is very high($\tau < \tau_M$), only foreign firm exists in the market.

Thus, with a non-negative constraint on royalty and fee, the following proposition then defines the optimal licensing contracts.

Proposition 1. With a non-negative constraint on royalty and fixed-fee, the optimal two-part licensing is following as

(a) $r^* = t$, $f^* = 0$ if $\tau \leq \frac{a-c-2t}{5}$,

(b) $r^* = \frac{a-c-5\tau}{2}$, $f(r^*) = \frac{(5\tau - a + 2t + c)(a - 2t - c + 7\tau)}{9}$ if $\frac{a-c-2t}{5} < \tau \leq \frac{a-c}{5}$,

(c) $r^* = 0$, $f(0) = \frac{4t(a+\tau-t-c)}{9}$ if $\tau > \frac{a-c}{5}$.

This proposition 1 implies that optimal royalty depends on levels of tariff and emission tax and it affects to determine the fixed-fee. Fig. 1 provides the optimal contracts, which are described by R(royalty) or T(two-part tariff), or F(fixed-fee). Then, it states that we can have either (i) royalty only (case a) or (ii) fixed fee only (case c). Otherwise, two-part licensing contracts have positive royalty and fixed fee (case b). In particular, if tariff and emission tax are lower levels, only royalty is optimal contract, $r^* > 0$. If those are moderate, two-part tariff licensing contract is optimal, $r^* > 0$ and $f(r^*) > 0$. But, if tariff level is high enough, fixed-fee licensing is only optimal.

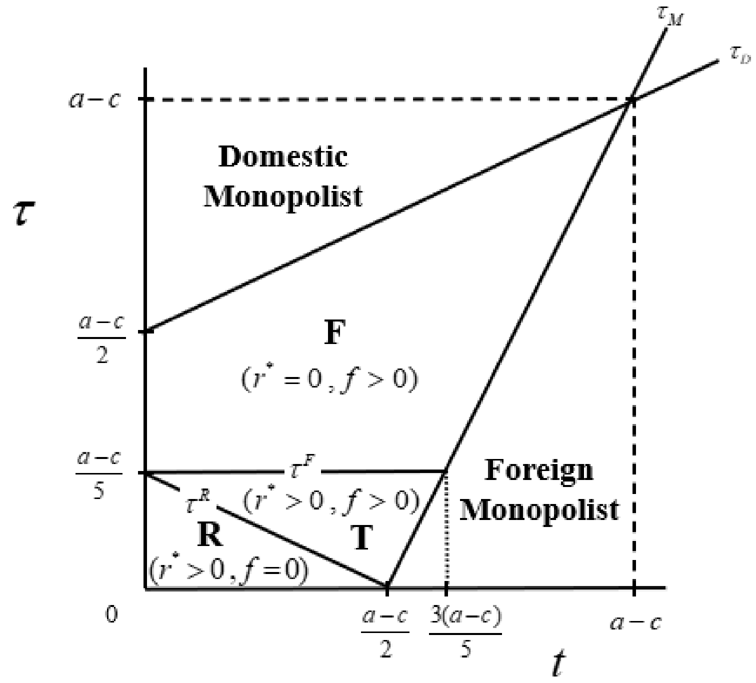

Fig. 1. Optimal licensing strategies of foreign firm with restriction r and f

Next, putting r^* and f^* into the profits of both firms in (8), we have the following profits under the optimal licensing contracts.

Under royalty licensing, the profits of the domestic firm and the foreign firm are as follows:

$$\pi_d^R = \left(\frac{a-c-2t+\tau}{3}\right)^2 \text{ and } \pi_f^R = \frac{(a-c)(a-c+5t-4\tau)+4\tau^2-t\tau-5t^2}{9} \tag{14}$$

Under pure fixed-fee licensing, the profit of a licensed domestic firm and a foreign firm are as follows:

$$\pi_d^F = \left(\frac{a-c-2t+\tau}{3}\right)^2 \text{ and } \pi_f^F = \frac{(a-c)(a-c+4t-4\tau)+4(\tau^2-t^2+\tau t)}{9} \tag{15}$$

Under two-part tariff licensing, the profit of a licensed domestic firm and a foreign firm are as follows:

$$\pi_d^T = \left(\frac{a-c-2t+\tau}{3}\right)^2 \text{and } \pi_f^T = \frac{(a-c)(5a-5c-26\tau)+41\tau^2+16t(a-c-t+\tau)}{36} \tag{16}$$

Note that the profit of domestic firm is the same under no licensing, i.e., $\pi_d^N = \pi_d^R = \pi_d^F = \pi_d^T$, but the domestic firm will accept the offered license by a foreign firm. This is because the foreign firm can construct two-part licensing contracts by taking the incentive compatibility constraint of the domestic firm and thus it can offer a favorable (discounted) contract to the domestic firm. On the other hand, we have the following relation that a foreign innovator has incentive to sell a license to domestic firm through all types of licensing: $\pi_f^R > \pi_f^N$, $\pi_f^F > \pi_f^N$ and $\pi_f^T > \pi_f^N$ for any τ and t.

Finally, we compare welfare functions under the two-part tariff licensing strategies in order to provide policy implications with regard to tariff policy and emission taxation. Under two-part licensing, the social welfare function can be defined by

$$W^L = \int_0^Q P(u)du - cq_d - pq_f + \tau q_f - rq_d - f$$

The resulting welfare from a licensing strategy with r and f is as follows, respectively:

$$W^R = W^R\{r^*, f = 0\} = \frac{1}{6}\left[2(a-c)(a-c-2t+\tau)+3(t^2-\tau^2)\right] \tag{17}$$

$$W^F = W^F\{r^* = 0, f(0)\} = \frac{1}{18}\left[6(a-c)(a-c+\tau)-8t(a+\tau-c-t)-9\tau^2\right] \tag{18}$$

$$W^T = W^T\{r^*, f(r^*)\} = \frac{1}{72}\left[17(a-c)^2+70\tau(a-c)-32t(a+\tau-c-t)-91\tau^2\right] \tag{19}$$

In order to focus on the welfare effect of the tariff, we suppose that $t = d$ where emission tax is based on the Pigouvian level in the below analysis. [4]

Proposition 2. In the case that $t = d$, we have (i) $W^R = W^N$, (ii) $W^T > W^N$, and (iii) $W^F \underset{<}{\overset{>}{-}} W^N$ if $\tau \underset{>}{\overset{<}{-}} \tau_w^*$ where $\tau_w^* = \dfrac{4(a-c)-t}{8}$ satisfies $W^F = W^N$.

Two-part licensing with a restrictive constraint ($r^* > 0$ and $f(r^*) > 0$) always improve welfare while royalty licensing with $r^* > 0$ cannot improve welfare as compared with non-licensing. However, fixed-fee licensing with $f(0) > 0$ can increase(decrease) welfare when $\tau < \tau_w^* (\tau > \tau_w^*)$. Thus, fixed-fee licensing may yield welfare losses with optimal choices of the domestic firm. Generally, increasing tariff affects to increase a domestic firm's profits and to

[4] We consider a Pigouvian tax on pollution damage level in the same as Chao *et al*. (2012) in order to focus on tariff effect. That is, we assumed that emission tax is exactly same with damage level.

decrease a foreign firm's profits. However, with existence of licensing strategy by a foreign innovator in domestic market competition, the domestic firm pay more fee to the foreign firm with increasing fee which is induced by either increasing emission tax or increasing tariff. To protect domestic industry by increasing tariff, it is rather than decreasing welfare by inducing lower output and higher price. Fig. 2 shows that the optimal choices of the domestic firm are not always socially desirable with response to tariff and emission tax expenditures under fixed-fee licensing. That is, there exists welfare losses. Comparing the welfares between fixed-fee licensing and non- licensing, we have $W^F < W^N$ in shaded areas in Fig. 2, in which non-licensing can induce higher welfare.

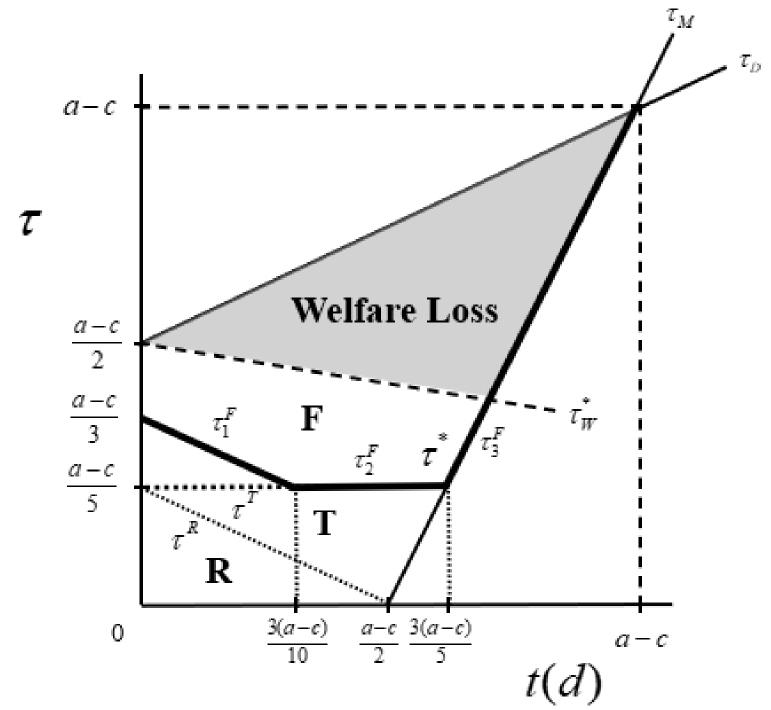

Fig. 2. Welfare losses and optimal tariff with restriction on r and f

In the previous analysis, we have found that the optimal decision of between a domestic firm and a foreign firm on two-part licensing depends on the level of tariffs and emission taxes. Therefore, under the strategic relationship among licensing contract, tariffs, and emission taxes, we will examine optimal tariff schedules and provide policy implications on the licensing strategies of eco-technology.

Proposition 3. The optimal tariff schedules of each licensing are following as

(i) Royalty licensing: $\tau^R = \dfrac{a-2t-c}{5}$ where $0 \le t < \dfrac{a-c}{2}$,

(ii) Two-part tariff licensing: $\tau^T \approx \dfrac{a-c}{5}$ where $0 \le t < \dfrac{3(a-c)}{5}$,

(iii) Fixed-fee licensing: $\tau_1^F = \dfrac{3(a-c)-4t}{9}$ where $0 \le t < \dfrac{3(a-c)}{10}$,

$$\tau_2^F = \frac{a-c}{5} \quad \text{where} \quad \frac{3(a-c)}{10} \le t < \frac{3(a-c)}{5}, \text{ and}$$

$$\tau_M = \tau_3^F = 2t + c - a \quad \text{where} \quad \frac{3(a-c)}{5} \le t < (a-c).$$

Fig. 2 shows that optimal tariff schedules bolded line τ^S where $S \equiv R, F, T$ (R: Royalty licensing, F: Fixed-fee Licensing, and T: Two-part tariff licensing). Under royalty licensing, when emission tax increases, the royalty increases and the outflow of profits to foreign firm increases, so the domestic welfare decreases. Therefore, tariffs must be levied as much as possible to offset this. Therefore, under royalty licensing, the optimal tariff has a negative relationship with the environmental tax. On the other hand, under two-part tariff licensing, the increase in emission tax reduces royalty and increases fee as much as the decrease of royalty, so for domestic welfare tariffs should be closely imposed to τ^S. Finally, under fixed-fee licensing, as the increase in emission tax, the optimal tariff first falls then constant and finally increases. In the areas where tariffs are relatively low, raising the environmental tax will increase the cost burden on a domestic firm and raising the tariff will also increase the amount of adjustments to be paid, so it is necessary to gradually reduce the tariff.

Lemma 1. Under optimal tax schedules, $W^{F*} > W^{T*} > W^{R*}$.

This lemma 1 states that welfare is maximized with optimal tariff under fixed-fee licensing which is higher than either that of royalty or two-part tariff licensing. This confirms a general finding that firm's profit is higher under two-part tariff licensing, while social welfare is greater under fixed-fee licensing [5]

Proposition 4. Under restrictive two-part licensing, the overall optimal tariff schedules by τ^* are as follows:

$$\tau^* = \begin{Bmatrix} \tau_1^F & if & 0 \le t < \dfrac{3(a-c)}{10} \\ \tau_2^F & if & \dfrac{3(a-c)}{10} \le t < \dfrac{3(a-c)}{5} \\ \tau_M & if & \dfrac{3(a-c)}{5} \le t < (a-c) \end{Bmatrix}$$

This proposition 4 implies that the optimal tariff rates should be chosen τ^* which induce fixed fee licensing to obtain maximizing the overall welfare. Fig. 2 shows the overall optimal tariff by heavy line. As the increase in emission tax, the optimal tariff first falls then constant and finally increases. There is a trade-off between reducing damaging effect and rent-leaking effect. Thus, if the government plans to make tighter emission tax policy, then he should lower the tariff to obtain maximizing welfare. Therefore, when both environmental taxes and tariffs are employed, these two policies may be strategic substitutes. [6]

6. Non-restrictive two-part licensing case with $r \gtreqless 0$ and $f \gtreqless 0$

Until now, we have analyzed two-part tariff licensing strategies with non-negative royalty and fixed-fee. However, it is possible for the patentee to consider the combination of a negative royalty and a negative fixed fee under two-part tariff licensing. Liao and Sen (2005) introduced subsidy in licensing with combinations if upfront fee and negative royalty. Therefore, in this section, we consider two-part licensing contract without a non-negative constraint on r and

[5] For example, Kabiraj and Kabiraj, (2017) showed that the government can impose a tariff under the two-part licensing contracts and induce fixed fee licensing of the foreign licensor to maximize domestic welfare. However, Mukherjee and Tsai (2014) showed that both firms and society may prefer two-part tariff licensing contract under costly technology transfer when the quality of licensed technology is endogenously chosen.

[6] Tsai et al. (2014) also showed that a relationship between tariffs and environmental taxes may be negative under the eco-technology when the government employs both policies.

f . Then, from same profit maximization problems for the foreign eco-innovated firm, we have the same optimal royalty as (11):

$$r^{*} = r = \frac{a-c-5\tau}{2} \tag{11}$$

Here, the difference of the previous analysis is that the optimal royalty can be positive or negative, i.e., $r \underset{<}{\overset{>}{-}} 0$ if $\tau \underset{<}{\overset{>}{-}} \frac{(a-c)}{5}$. Then, we also have the same optimal fixed-fee f^{*} in (12) which can be even negative. Thus, we have the same with τ^{R} and τ^{F} in (13). Then, Fig. 3 provides the relations of the findings where τ^{R} and τ^{F} are the boundaries of duopolistic competition.

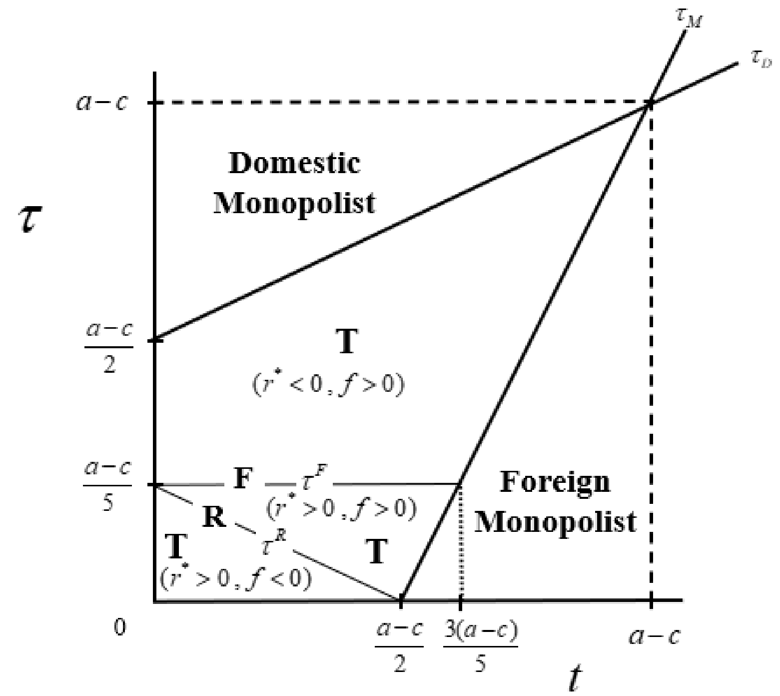

Fig. 3. Optimal licensing strategies of foreign firm with non-restriction on r and f

Proposition 5. Foreign eco-innovated firm's optimal two-part licensing schemes without non-negativity constraints are as follows:

(a) $r^{*} = t$, $f^{*} = 0$ if $\tau = \tau^{R}$,

(b) $r^* = 0$, $f(0) = \dfrac{4t\{6(a-c)-5t\}}{45}$ if $\tau = \tau^F$,

(c) $r^* = \dfrac{a-c-5\tau}{2} > 0$, $f(r^*) = \dfrac{(5\tau - a + 2t + c)(a - 2t - c + 7\tau)}{9} < 0$ if $\tau < \tau^R$,

(d) $r^* = \dfrac{a-c-5\tau}{2} < 0$, $f(r^*) = \dfrac{(5\tau - a + 2t + c)(a - 2t - c + 7\tau)}{9} > 0$ if $\tau > \tau^F$,

(e) $r^* = \dfrac{a-c-5\tau}{2} > 0$, $f(r^*) = \dfrac{(5\tau - a + 2t + c)(a - 2t - c + 7\tau)}{9} > 0$ if $\tau^R < \tau < \tau^F$.

Proposition 5 provides the optimal contracts, which are described by R(royalty) or T(two-part tariff), or F(fixed-fee). Then, it states that we can have either (i) royalty only (case a) or (ii) fixed fee only (case b). Further, we also have either (i) two-part licensing with negative royalty (case d) or (ii) two-part licensing with negative fixed fee (case c). Otherwise, two-part licensing contracts have positive royalty and fixed fee (case e).

Proposition 5 also implies that optimal licensing strategies depend on levels of tariff and emission tax. In particular, if tariff is moderate between $\tau^R < \tau < \tau^F$, the two-part licensing contract with positive royalty and fixed fee is optimal. Otherwise, non-negative constraints are essential to construct two-part licensing contracts.

On the one hand, if tariff and emission tax are lower levels, i.e., $\tau < \tau^R$, the optimal royalty is always positive while the optimal fixed fee will be negative. Note that if we impose a non-negative constraint of fixed fee, only royalty is an optimal contract, i.e., $r^* > 0$ and $f^* = 0$. In such a case that the government policy is soft to both firms, their competition becomes a typical licensing model of internal innovation between the firms with asymmetric cost. [7] However, if the non-negative fixed fee is available, a lower tariff raises the royalty and reduces the fixed fee to a negative value. This is because the foreign firm can induce the domestic firm to take a contract by providing a negative fixed fee (subscription subsidy) and then impose

[7] In a standard patent licensing with internal innovation, it is shown that royalty licensing is preferred to fixed-fee licensing in a Cournot duopoly with a homogeneous good (Wang, 1998), a differentiated Bertrand duopoly (Wang & Yang, 1999), incumbent innovators in a homogeneous Cournot oligopoly (Kamien & Tauman, 2002), and a leadership structure (Kabiraj, 2005). However, Wang (2002) shows that fixed-fee licensing is preferred to royalty licensing for an internal patentee with a heterogeneous duopoly if the product differentiation is sufficiently large.

royalty in order to have relative cost advantage, which will return back its loss from the subsidy to its ex post profit.

On the other hand, if tariff is high, i.e., $\tau < \tau^F$, the optimal fixed fee is always positive while the optimal royalty will be negative. Note that if we impose a non-negative constraint of royalty, only fixed fee is an optimal contract, i.e., $r^* = 0$ and $f^* > 0$. Thus, if the tariff is hard to the foreign firm to enter the domestic market, a higher tariff raises fixed fee and reduces royalty to a negative value. This is because the foreign firm can induce the domestic firm to take a contract by providing a negative royalty (usage subsidy), which yields its rival to have cost advantage and increase outputs, but impose a higher fixed fee in order to retrieve the increased profit of the domestic firm into its profit. Note that this is contrast to the results of Kabiraj and Kabiraj (2017) and Yang *et al*. (2020) who imposed a non-negative royalty and showed that fixed-fee licensing could dominate two-part licensing when import tariff rate is high.

Next, from Eqs. (4) and (16), we have the following relation that a foreign innovator has incentive to sell a license to domestic firm through non-restrictive two-part licensing: $\pi_f^T > \pi_f^N$ for any τ and t. It is easy to verify that $\pi_f^T - \pi_f^N = \dfrac{(a-c-5\tau)^2 + 4t\,[2(a-c)-5t+8\tau]}{36} > 0$.

Note that the profit of domestic firm is the same under no licensing, i.e., $\pi_d^N = \pi_d^T$, but the domestic firm will accept the offered license by a foreign firm. This is because the foreign firm can construct two-part licensing contracts by taking the incentive compatibility constraint of the domestic firm and thus it can offer a favorable (discounted) contract to the domestic firm.

Finally, we examine and compare the welfares between non-licensing and two-part licensing strategies from Eqs. (5) and (19).

Proposition 6. In the case that $t = d$, we have (i) $W^T < W^N$ if either $\tau > \tau_W^{**}$ or $\tau < \tau^R$, (ii) $W^T \geq W^N$ if $\tau^R \leq \tau \leq \tau_W^{**}$ where $\tau^R = \tau_W^{***} = \dfrac{a-2t-c}{5}$ and $\tau_W^{**} = \dfrac{7(a-c)-2t}{11}$ satisfies $W^T = W^N$.

Proposition 6 states that two-part licensing might improve the welfare if the tariff is high enough, but yield welfare loss otherwise. That is, if the tariff is either very high or low, welfare-improving effect (from the elimination of emissions) under the licensing will be outweighed by the welfare-distorting effect (from rent-leakage effect to the foreign firm).

Next, we consider the first stage where the government determines the optimal tariff. Then, from the first-order condition of (19), we have that $\frac{\partial W^T}{\partial \tau} = \frac{35(a-c)-16t-91\tau}{36}$ =0. Then, we can obtain the optimal tariff schedule τ^T as follows:

$$\tau^T = \frac{35(a-c)-16t}{91} \tag{22}$$

***Proposition** 7*. Under non-restrictive two-part tariff licensing, the overall optimal tariff schedules by τ^{**} are as follows:

$$\tau^{**} = \begin{cases} \tau^T & if \quad 0 \le t < \dfrac{7(a-c)}{11} \\ \tau_M & if \quad t \ge \dfrac{7(a-c)}{11} \end{cases}$$

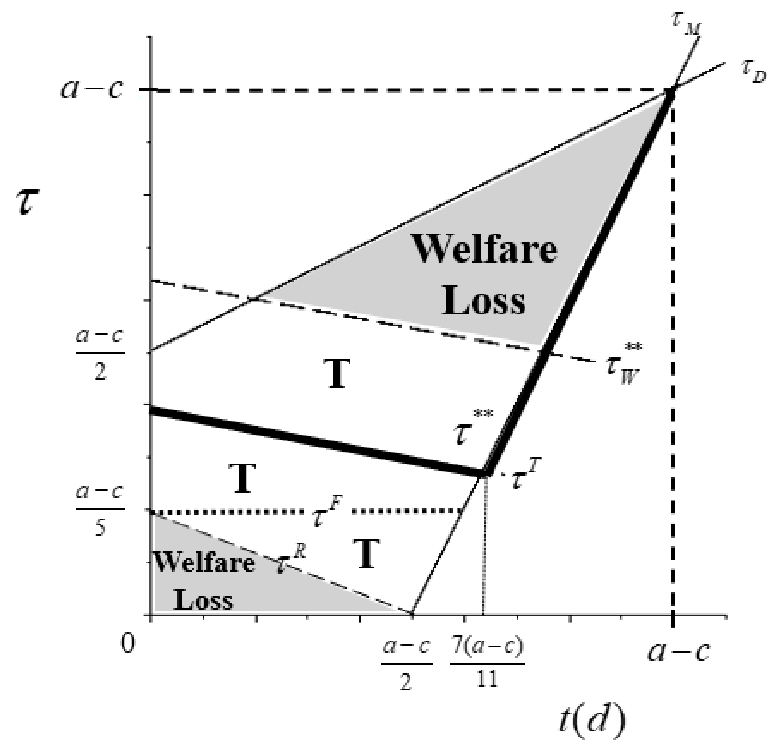

Fig. 4. Welfare losses and optimal tariff with non-restriction on r and f

Fig. 4 shows the comparisons between τ^{**} and τ_W^{**}. First, it shows that $\tau^F < \tau^{**} < \tau_W^{**}$ for any t. This implies that when the government imposes the optimal tariff, the foreign firm implements two-part licensing contract with a negative royalty and a positive fixed fee, which always improves the domestic welfare, compared to the no licensing. Second, it is shown that τ^{**} decreases as the emission tax increases, i.e., $\frac{\partial \tau_T^*}{\partial t} < 0$. This implies that the strategic policy relationship between trade policy and environmental policy could be negative. This is because the eco-technology innovation can induce the government can replace emission tax with tariff policy. For example, if the environmental damage is serious and thus the emission tax rate is high, it is desirable to eliminate the welfare loss from the pollutants. In that case, the optimal policy choice of tariff should be low in order to induce the eco-foreign firm to enter the domestic market with cost advantage when eco-technology licensing contract is forthcoming. [8] However, if the environmental damage is not serious and thus the emission tax rate is low, it is desirable to make domestic firm be with cost advantage. In that case, rent leakage effect is serious than pollution effect and thus the optimal policy choice of tariff should be higher when eco-technology licensing contract is forthcoming.

Proposition 8. The optimal tariff schedule τ^{**} can improve domestic welfare when the two-part licensing contract is forthcoming, but it has negative relation with emission tax.

7. Comparison: restrictive vs. non-restrictive two-part licensing

In this section, we compare the result of between restrictive and non-restrictive two-part licensing contracts. First, we examine whether the foreign innovated competitor has a strategic incentive to choose either a negative royalty or a negative fixed fee under two part licensing. Then, we compare the profit functions of two cases from Eqs. (14)-(16), then, we get

[8] It is known that the traditional policy relation between the environmental tax and tariff under imperfect competition is positive because there exists a trade-off between environmental damage and allocative inefficiency. See Chao et al. (2012). Hence, when the environmental damage is serious, then the government should impose higher emission tax to the polluting domestic firms while it should raise the tariff since domestic firms have cost disadvantage from the emission tax, which might increase the rent-extraction effect from the foreign competitors. However, when the environmental damage is not serious, then the government may impose lower emission tax while it can reduce the tariff in order to increase market competition. For more discussion, see ***. However, if the technology innovation and eco-R&D issues are involved, the policy relation might be reversed. See *** Tsai, MORE discussion will be added.

$\pi_f^{T**} - \pi_f^{F*} = \dfrac{(a-c-5\tau)^2}{36} > 0$ and $\pi_f^{T**} - \pi_f^{R*} = \dfrac{(a-c-2t-5\tau)^2}{36} > 0$ for any τ and t where $\pi_f^{T**} \equiv \pi_f^T(r \gtrless 0 \;\&\; f \gtrless 0)$, $\pi_f^{F*} \equiv \pi_f^F(\tau^F)$, and $\pi_f^{R*} \equiv \pi_f^R(\tau^R)$. This implies the following proposition.

Proposition 9. Foreign competitor prefers non-restrictive two-part licensing to restrictive licensing.

Next, we examine the effect of welfare between the two cases. From proposition 2 and 6, we get that $\tau_w^{**} - \tau_w^{*} = \dfrac{3(a-c)+7t}{22} > 0$ for all t. Then, we have the following welfare implications.

Proposition 10. Under non-restrictive two-part licensing contract, welfare loss is lessen where $\tau_w^{*} < \tau_w^{**}$ while welfare loss arises in the ranges where $\tau < \tau^R$.

This proposition implies that the range of welfare losses can be reduced in the absence of restriction of two-part tariff licensing. It is possible to obtain more profits by subsidizing first then taking them as fixed rather than taking them all with only fixed fee, and by giving subsidies in the domestic market, it is possible to produce more products and expand the range of two-part tariff licensing with the negative royalty(subsidy). However, if tariffs and environmental taxes are very low, it may cause welfare losses under non-restrictive case. This means that royalty licensing of restrictive two-part licensing is better to the society than two-part tariff licensing with non-restrictive assumption.

Proposition 11. Under non-restrictive two-part licensing contract, the optimal tariff schedule τ^{**} can improve welfares.

The above proposition implies that when the optimal tariff is imposed, welfare is higher under non-restrictive two-part tariff licensing than under restrictive two-part tariff licensing on the royalty and the fixed-fee. That is, the government may consider strategic choice of foreign firm with a negative royalty (subsidy) to prevent loss of welfare and then impose a higher level of tariffs τ^{**} than τ^{*} to improve social welfare (See Fig. 5).

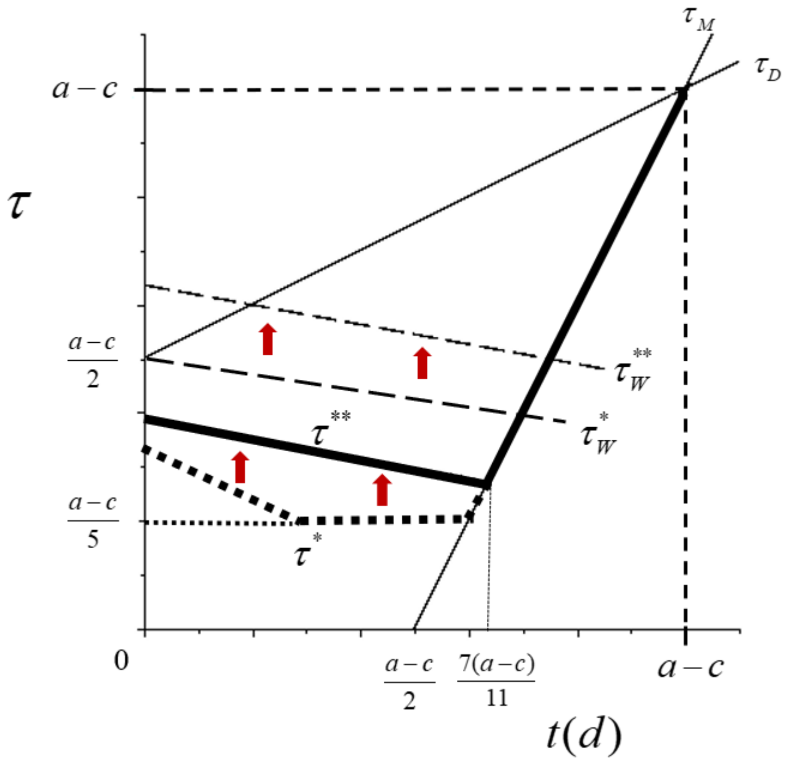

Fig. 5. The overall optimal tariffs of restrictive vs. non-restrictive two-part licensing

8. Conclusion

We examined the two-part licensing contracts of eco-technology by a foreign-innovator which competes with a polluting domestic firm in a home country, and investigated the government's optimal tariff policies facing with an emission tax. We considered and compared the two-part licensing contracts with and without non-negative royalty and fixed-fee, and showed that the foreign eco-innovator will choose the non-restrictive two-part licensing contracts with a negative royalty or a fixed fee, depending on the levels of emission tax and tariff. We also showed that the non-restrictive two-part licensing contract is better off to the domestic welfare than that with restrictive two-part licensing contract. Finally, we showed that the optimal tariff policy under the non-restrictive licensing contract has a negative relation with emission tax.

In the future research, the possible extensions are to analyze trade policy and emission regulation with the leader-follower model and the asymmetric cost model in the international duopoly.

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Appendix

Proof of Lemma 1.

From optimal tariff schedule of proposition 6, we can derive the following welfares:

$$W^{R*} \equiv W^R(\tau_R) = \frac{19(a-c)^2}{50} + t\left(\frac{21}{50}t - \frac{18(a-c)}{25}\right) \tag{A1}$$

$$W^{T*} \equiv W^T(\tau_T) = \frac{19(a-c)^2}{50} + \left(\frac{4}{9}t - \frac{8(a-c)}{15}\right) \tag{A2}$$

$$W^{F*} \equiv W^F(\tau_1^F) = \frac{7(a-c)^2}{18} + t\left(\frac{44}{81}t - \frac{16(a-c)}{27}\right) \tag{A3}$$

We have $W^{T*} - W^{R*} = \frac{t}{450}\left(84(a-c)+11t\right) > 0$ from Eqs. (A1) and (A2) and $W^{F*} - W^{T*} = \frac{2(3a-3c-10t)^2}{2025} > 0$ from Eqs. (A3) and (A2). Thus, $W^{F*} > W^{T*} > W^{R*}$.■

Proof of Proposition 4.

From lemma 1, we have the overall optimal tariffs which are (i) τ_1^F if $0 \le t < \frac{3(a-c)}{10}$ since $W^{F*} > W^{T*} > W^{R*}$, (ii) τ_2^F if $\frac{3(a-c)}{10} \le t < \frac{3(a-c)}{5}$ since $W^{F*} = W^{T*} > W^{R*}$, and (iii) τ_M if $t \ge \frac{3(a-c)}{5}$ since $W^{F*} > W_F^M$ where $W_F^M = \frac{3}{8}(a-c-\tau)^2$ which is the welfare of foreign monopoly case.■

Proof of Proposition 11.

We get that

(i) $W^T(\tau^{**}) - W^F(\tau^* = \tau_1^F) = \frac{4(a-c)^2}{117} - \frac{8t[21(a-c)+55t]}{7371} > 0$ for $0 \le t < \frac{3(a-c)}{10}$,

(ii) $W^T(\tau^{**}) - W^F(\tau^* = \tau_2^F) = \frac{2[21(a-c)-20t]^2}{20475} > 0$ for $\frac{3(a-c)}{10} \le t < \frac{3(a-c)}{5}$, and

(iii) $W^T(\tau^{**}) - W^F(\tau^* = \tau_3^F) = \frac{12(a-c)^2}{13} - \frac{2t[1344(a-c)+1199t]}{819} > 0$ for $0 \le t < a-c$.■

Usage Lock-In and Platform Competition under Multihoming*

Preliminary draft, comments welcome

Susumu Sato†

April 12, 2021

Using a competition-in-utility framework, I analyze a model of platform competition where consumers choose which platform to use for completing a bundle of transactions with sellers, which I call bundled usage. I show that, compared to the case where consumers can choose which platform to use for each transaction, bundled usage leads to a greater per-transaction consumer surplus, reflecting the platforms' incentives to divert transactions from other channels. Compared to the socially optimum level, the equilibrium consumer utility is too high. Analyses of entry, merger, and limit pricing show that platform competition often exacerbates the excessive consumer utility, adversely affecting welfare. Finally, allowing for mixed-homing, I show that a higher multihoming cost of consumers lowers the equilibrium consumer utility, indicating that multihoming is key driver of the excessive competition for usage. The result of this study provides a rationale for policies that restrict platforms' strategies that benefit consumers at the expense of sellers.
Keywords: Two-sided markets; multihoming; competition-in-utility
JEL Codes: L13; L81

1 Introduction

Two-sided platforms such as online marketplaces, content streaming services, and operating systems provide market participants with transaction opportunities. In such markets, participants such as consumers and sellers often join multiple platforms (i.e., multihome) and use the different platforms in different situations. As a result, platforms face the competition for usage as well as the competition for membership. For example, a consumer

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survey shows that in Japanese e-commerce market, although Amazon has the largest share in terms of the membership, Rakuten Ichiba has the larger share in terms of the number of usage of the marketplace, which indicates that the margin of the competition for usage differs from that of competition for membership.[1] This study aims at analyzing the competition for usage among platforms under multihoming environments.

Consumers' patterns of platform usage may substantially differ depending on markets. For example, in the payment industry, a consumer typically visits a merchant and then choose which payment method to use. In this case, consumers choose the platform for each transaction with sellers. I call this pattern of usage as *separate usage*. Alternatively, in online marketplaces, consumers may have a bundle of transactions in mind and choose which platform to use for completing those transactions. I call this pattern of usage as *bundled usage* and the situation in which the usage is bundled as *usage lock-in*.

Usage lock-in can take several forms. First, consumers often choose a particular platform to complete a large portion of transactions with sellers. For example, consumers often select which marketplace to use to buy certain sets of products as described as "one-stop shopping" (Armstrong and Vickers, 2010; Thomassen, Smith, Seiler and Schiraldi, 2017). This pattern of usage may arise from shopping costs, switching costs, or loyalty programs designed by marketplaces. Similarly, in software markets, consumers often choose the "main" software platform, such as operating system, to install applications on top of that platform, as typically observed in the discussion of the "browser war" between Microsoft and Netscape (Gilbert and Katz, 2001).

To understand the consequence of platform competition for usage and usage lock-in, I analyze the competition among platforms for usage when consumers and sellers multihome various platforms. In particular, to incorporate various non-monetary design choices made by the platform, I adopt a competition-in-utility framework (Armstrong and Vickers, 2001) of platform competition with the following features:

1. There are ex-ante identical consumers and sellers that can join any combination of the platforms. After observing the preference for usage, consumers choose which platform to use, among those they have joined, for completing the bundle of transactions with users.

2. Consumers and sellers can transact through a direct channel without joining platforms.

3. Platforms set the level of per-transaction consumer utilities u_i and charge membership fees (P_i, T_i) to consumers and sellers.

The equilibrium membership fees are characterized by the incremental-value pricing as observed in the literature of advertising platforms (Anderson, Foros and Kind, 2018).

[1] MarkeZine (in Japanese), https://markezine.jp/article/detail/33787.

Platforms set membership fees that equal the incremental values that consumers and sellers obtain by adding each platform into the portfolio of platforms to join. With this incremental-value pricing, platforms choose the level of consumer utilities to maximize their incremental values plus their own transaction profits.

Given this characterization of equilibrium membership fees, I analyze the properties of the consumer utilities set by platforms. First, to understand the implication of bundled usage, I compare the equilibrium consumer utility with the equilibrium consumer utility under *separate usage*: the case where consumers can choose which platform to choose for completing each transaction with a seller. I show that usage lock-in intensifies the competition; the equilibrium consumer utility under bundled usage is higher than under separate usage. This is because platforms have a stronger incentive to divert consumer usage from other channels to increase sellers' incremental values under bundled usage. Next, I show that the equilibrium consumer utilities under bundled usage are too high from the welfare viewpoint. This welfare result arises from the divergence between the welfare gain from raising consumer utilities and the increases in the incremental values. From the consumer's perspective, the increase in the platform's incremental value equals the increase in consumer surplus. However, from the seller's perspective, although the increase in the probability of usage of one platform increases the incremental value of that particular platform, the welfare gain is smaller because such an increase in the probability of usage partly replaces the probability of the usage of other platforms or direct channels. Put differently, platforms have an excessive incentive to divert transactions from other channels. As a result, platforms set too high consumer utilities to increase the revenues from sellers.

The fact that platforms set inefficiently high consumer utilities suggests that intensifying competition might harm welfare. To examine this possibility, I analyze entry, mergers, and limit pricing as changes in competitive environments. In the analysis of entry, I show that welfare may be U-shaped in the number of platforms; that is, an entry of a platform may be harmful to welfare when the number of platforms is small. Next, in the merger analysis, I show that the merger to monopoly mitigates the excessively high consumer utility and improves welfare. Finally, in the analysis of limit pricing, I show that when entry deterrence occurs, the resulting welfare is lower than that without the threat of entry. In total, in the environment with bundled usage and multihoming, naive promotion of competition may harm welfare.

Finally, to understand the importance of consumer multihoming for excessively high consumer utilities and the robustness of main results, I extend the model in two directions. First, I introduce the heterogeneity in consumers' membership utilities and the multihoming costs so that singlehoming and multihoming consumers co-exist. Then, I show that an increase in multihoming cost reduces the level of equilibrium consumer utility, which indicates that multihoming is the crucial driving force for obtaining excessive consumer utility. Second, I introduce the heterogeneity in the trade surplus of sellers so that equilibrium consumer utility tends to be too low due to Spence distortion (Weyl, 2010). With this extension, I show that equilibrium consumer utility can be either too low or too high depending on the relative size of Spence distortion.

The results of this study have several policy implications. First, when consumer multihoming is relevant, usage is bundled, and there are ample non-proprietary platforms that play a role of direct channels, then regulatory agencies should be cautious about the inefficiently generous offers made by platforms to consumers such as low transaction fees, coupons, and free add-ons. This implication gives a novel rationale for Japanese competition policy in online marketplaces. Japan Fair Trade Commission (JFTC) filed a petition for an urgent injunction against "Shipping Inclusive Program Measures" proposed by Rakuten, Inc., by which all the sellers of "Rakuten Ichiba" would be prevented from receiving delivery fees from consumers.[2] JFTC also intervened with the Amazon Japan's expansion of reward program that required the sellers to pay the cost for rewards.[3] The result of the present study provides a novel rationale for intervening with these seemingly consumer-benefiting but seller-harming platform designs. Second, when excessively consumer-friendly platform designs are detrimental to the welfare, naive promotion of platform competition may have adverse effects. Instead, fostering a shift from bundled usage to separate usage by reducing shopping costs would be a better policy alternative.

2 Related literature

This study is related to the studies of the platform competition under multihoming (Ambrus, Calvano and Reisinger, 2016; Anderson et al., 2018; Bakos and Halaburda, forthcoming; Calvano and Polo, 2020). These studies show that when both sides of participants multihome, platforms do not directly compete for membership (Anderson et al., 2018), and sometimes two-sided pricing characterized by cross-subsidization breaks down (Bakos and Halaburda, forthcoming). In the present study, although the platforms do not directly compete for the membership, they compete for the usage. The competition for the usage restores the two-sided pricing even in the presence of multihoming on both sides. Further, such competition for usage leads to excessively high consumer utilities.

Another strand of related literature is platform competition for usage (Rochet and Tirole, 2003; Liu, Teh, Wright and Zhou, 2019). These studies analyze the platform competition for usage, assuming that platforms charge only transaction fees. This modeling feature is suited to markets such as ride-sharing, payment cards, and online travel

[2] Japan Fair Trade Commission, https://www.jftc.go.jp/en/pressreleases/yearly-2020/February/200228.html

[3] Nikkei Asia, https://asia.nikkei.com/Business/Companies/Japan-s-antitrust-regulator-to-probe-Amazon-s-loyalty-program

agencies. Compared to this literature, the present study allows platforms to charge membership fees. This modeling feature is suited to markets such as content streaming services, some types of online marketplaces, and software markets in which subscription fees or the prices for devices are relevant strategic instruments.

This study is also related to the study of the welfare effect of platforms in the presence of direct channels (Edelman and Wright, 2015). Edelman and Wright (2015) show that when platforms use price parity clauses, the platform invests too much in consumer benefit from platform usage, and too many consumers use the platform in the equilibrium. Although the result of excessive intermediation is similar, the driving force of excessive intermediation and its implication is different in the present study. In Edelman and Wright (2015), the presence of price parity clauses increases product prices at direct channels, which plays a major role in generating excessive intermediation results by reducing the surplus of consumers outside the platform. In this study's framework, platforms set membership fees that equal the incremental values, and each platform can increase its incremental value for sellers by increasing the probability that consumers use the platform. However, this increase in the incremental value for sellers does not contribute to the welfare, as it just replaces the value for sellers between other channels, which is the source of excessive intermediation in the present study. As a result, even without price parity clauses, excessive intermediation arises.

Finally, this study contributes to the body of research that studies the impact of competition in multi-sided markets, including entry (Liu et al., 2019; Tan and Zhou, 2020) and mergers (Correia-da Silva, Jullien, Lefouili and Pinho, 2019; Anderson and Peitz, 2020; Sato, 2020). A major contribution of the present study to this literature is the finding of a novel mechanism under which competition reduces welfare in two-sided markets. In the present study, competition often hurts welfare due to already excessively high equilibrium consumer utilities. This mechanism is totally different from the trade-off between the welfare benefit of competition and the welfare cost of shrinking network sizes.

3 Model

In this section, I propose two models of platform competition. The first one is the main model of *bundled usage*, where consumers must choose which platform to use for completing transactions with all the sellers. The second one is a model of *separate usage*, where consumers can choose which platform to use for completing each transaction with each seller.

There is a unit mass of consumers and sellers in a market. Also, there are proprietary platforms and a non-proprietary platform that enable the trades between consumers and sellers. Let $\mathcal{N}$ be the finite set of proprietary platforms with $|\mathcal{N}| = n$. The non-proprietary platform is viewed as direct sales channels such as direct-to-consumer sales or open-source operating systems such as Linux. Proprietary platforms set per-transaction utilities of consumers $(u_i)_{i\in\mathcal{N}}$ and charge membership fees $(P_i, T_i)_{i\in\mathcal{N}}$ to consumers and sellers.

consumers and sellers All consumers and sellers can freely trade on non-proprietary platform labelled as platform 0, in which case, consumers and sellers obtain the surplus $u_0 > 0$ and $v_0 > 0$, respectively. When a consumer uses a proprietary platform $i \in \mathcal{N}$, she trades with sellers on that platform, in which case, a consumer, a seller, and the platform obtain the surplus $u_i \in [\underline{u}, \overline{u}] \subset \mathbb{R}_{++}$, $v(u_i) \in \mathbb{R}_+$, and $\pi(u_i) \in \mathbb{R}$, respectively. Each consumer also derives a idiosyncratic benefit ϵ_i from the transaction on platform i. I assume that ϵ_i follows a distribution function $F(\epsilon_i)$ that is i.i.d. across consumers and platforms and has support $[\underline{\epsilon}, \overline{\epsilon}]$ and continuous density function $f(\epsilon_i)$. I also assume that $1 - F$ is log-concave. Further, for an analytical ease, I assume that $\overline{\epsilon} = \infty$ and $\underline{\epsilon} + u + v(u) + \pi(u) < u_0$ for all $x \in [\underline{u}, \overline{u}]$. Given a vector $\mathbf{u} = (u_1, u_2, \ldots, u_N)$ consumer utilities set by platforms, when a consumer joins the set $\mathcal{P} \subset \mathcal{N}$ of proprietary platforms and sellers join the set $\mathcal{S}$, she observes the preference for each platform $\epsilon = (\epsilon_i)_{i\in\mathcal{P}\cap\mathcal{S}}$. Then, each consumer uses the platform that maximizes the expected utility.

Let $w(u) := u + v(u) + \pi(u)$ be the total surplus made by each trade. I put the following restrictions on the functions v and π.

Assumption 1. *The functions v and π satisfy the following properties:*

1. *v and π are weakly concave;*
2. *There exists $\hat{u} > \underline{u}$ and $\widetilde{u} \in [\hat{u}, \overline{u})$ such that $w'(\hat{u}) = 0$ and $v(\widetilde{u}) + \pi(\widetilde{u}) = 0$.*

This assumption implies that $w(u)$ is weakly concave and has maximum in $(\underline{u}, \overline{u})$. In addition, if per-transaction joint surplus of sellers and platform is negative, a further increase in consumer utility lowers the per-transaction total surplus.

Let $U^{\mathcal{P}\cap\mathcal{S}}(\mathbf{u})$ be the ex-ante expected surplus from trade when a consumer joins the set $\mathcal{P}$ of proprietary platforms and sellers join the set $\mathcal{S}$ of proprietary platforms. The exact form of $U^{\mathcal{P}\cap\mathcal{S}}(\mathbf{u})$ is given by

$$U^{\mathcal{P}\cap\mathcal{S}}(\mathbf{u}) = \mathbb{E}_\epsilon \left\{ \max_{i\in\mathcal{P}\cap\mathcal{S}} \left[\max\{u_0, u_i + \epsilon_i\} \right] \right\} \tag{1}$$

Each consumer chooses the portfolio of the platforms that she joins to maximize the expected surplus from trade minus the membership fees $(P_i)_{i\in\mathcal{N}}$, which is written as the following problem,

$$\max_{\mathcal{P}\in 2^{\mathcal{N}}} U^{\mathcal{P}\cap\mathcal{S}}(\mathbf{u}) - \sum_{i\in\mathcal{P}} P_i, \tag{2}$$

where $\mathcal{S}$ is the set of platforms that sellers join.

Next, consider the sellers' payoff from choosing a portfolio of platforms that they join. Each seller's payoffs from the participation choice under bundled usage and separate usage differ. Under the bundled usage, each seller cannot affect consumers' usage probability of platforms, because each seller is atomless. By contrast, under the separate usage, each seller can affect consumers' usage probability of platforms. Let $d_i^{\mathcal{P}}(\mathbf{u})$ be the probability that a consumer uses the platform $i \in \mathcal{N} \cup \{0\}$ when the consumers join the set $\mathcal{P}$ of platforms. Then, under bundled usage, each seller joining the set $\mathcal{S}$ of platforms yields the surplus

$$V^{\mathcal{P},\mathcal{S}}(\mathbf{u}) = d_0^{\mathcal{P}}(\mathbf{u})u_S^0 + \sum_{i \in \mathcal{P} \cap \mathcal{S}} d_i^{\mathcal{P}}(\mathbf{u})v(u_i) \tag{3}$$

Under separate usage, each seller joining a set $\mathcal{S}$ of platforms yields the surplus

$$V^{\mathcal{P}\cap\mathcal{S},\mathcal{S}}(\mathbf{u}) = d_0^{\mathcal{P}\cap\mathcal{S}}(\mathbf{u})u_S^0 + \sum_{i \in \mathcal{P} \cap \mathcal{S}} d_i^{\mathcal{P}\cap\mathcal{S}}(\mathbf{u})v(u_i) \tag{4}$$

Thus, given the consumer utilities $(u_i)_{i \in \mathcal{N}}$ and membership fees $(T_i)_{i \in \mathcal{N}}$, sellers choose the set of platforms $\mathcal{S}$ to join to maximize

$$V^{\mathcal{P},\mathcal{S}}(\mathbf{u}) - \sum_{i \in \mathcal{S}} T_i \tag{5}$$

under bundled usage and

$$V^{\mathcal{P}\cap\mathcal{S},\mathcal{S}}(\mathbf{u}) - \sum_{i \in \mathcal{S}} T_i \tag{6}$$

under separate usage.

Platforms Each platform i obtains profit $\pi(u_i)$ for each transaction. Thus, if consumers and sellers join the sets $\mathcal{P}$ and $\mathcal{S}$ of platforms respectively, and consumers use the platform i with probability $d_i^{\mathcal{P}\cap\mathcal{S}}$, the profit of platform i is given by

$$\Pi^{\mathcal{P},\mathcal{S}}(\mathbf{u}) = 1_{i \in \mathcal{P}} P_i + 1_{i \in \mathcal{S}} T_i + d_i^{\mathcal{P}\cap\mathcal{S}}(\mathbf{u})\pi(u_i), \tag{7}$$

where 1_X is an indicator function that takes the value 1 if X is true and 0 if X is false.

Timing The timing of the game is given as follows:

1. Platforms $i \in \mathcal{N}$ simultaneously set consumer utilities $(u_i)_{i \in \mathcal{N}}$.

2. Observing the profile of consumer utilities, platforms simultaneously set membership fees $(P_i, T_i)_{i \in \mathcal{N}}$

3. consumers and sellers choose the set of platforms to join.

4. Given the sets of the platforms consumers and sellers joined and the realization of ϵ, each consumer chooses which platform to use in the following manner:
   - In the case of bundled usage, each consumer chooses which platform to use for making transactions with all the sellers on that platform.
   - In the case of separate usage, each consumer chooses which platform to use for making a transaction with each seller.

Equilibrium concept I adopt subgame-perfect equilibrium as a solution concept. As in the standard literature of two-sided markets, there are multiple equilibria in consumers' and sellers' participation decisions. To analyze the relevant cases where all platforms are active, I focus on the symmetric equilibria where all consumers and sellers join all the platforms. I call such equilibria as *participation equilibria*. In the next section, I characterize the participation equilibria and show their welfare properties.

Discussion on the modeling assumption This model has two important assumption worth discussing. Under the separate usage, consumers choose which platform to use for each transaction with each seller. This setting is similar to Liu et al. (2019)'s multihoming setting *given* the set of platforms to join. Under the bundled usage, each consumer choose which platform to use for all transactions with sellers. This setting is again similar to Liu et al. (2019)'s singlehoming setting given the set of platforms to join. The bundled usage setting of this study can be viewed as the situation where consumers choose which online marketplace to visit to purchase a bundle of products, which typically fits the purchasing behavior in grocery stores, as discussed by Armstrong and Vickers (2010), Zhou (2014) and Thomassen et al. (2017). Similarly, this interpretation is applicable to content-streaming services. One important difference between the current setting and Liu et al. (2019) is that platforms can charge membership fees such as subscription fees or the prices for computers in the current setting. Consumers often subscribe multiple music- or video-streaming platforms such as Spotify, Apple Music, Amazon Prime Music, Netflix, Hulu, Amazon Prime Video, etc. In this example, direct channels can be interpreted as offline stores selling physical media such as CDs and DVDs. Another interpretation is that consumers decide which platform to use as the main platform on which consumers trade with sellers. For example, consumers may purchase several PCs, tablets, and smartphones and uses some of them as the main OS to use application software, which might include non-proprietary OS such as Linux. In this interpretation, several platforms are consumed by consumers as genuine platforms to transact with sellers, and others are consumed just like products. Although it seems that the assumption of bundled usage is restrictive, it is not as it seems at first sight. The crucial difference between bundled usage and separate usage is that each seller is non-atomic for consumers under bundled usage. Indeed, what we need to derive the results under bundled usage is to assume that there exists a positive mass of transactions that consumers bundle. Suppose that each consumer has $m > 0$ categories and that the consumer chooses which platform to use for completing all the transactions with sellers in each category. Then, it can be shown that the equilibrium is the same as the model of bundled usage.

Secondly, in this study I assume that platforms can charge lump-sum membership fees (P_i, T_i) to consumers and sellers. This assumption is plausible in certain environments where subscription fees or prices for devices are relevant, but not in others where there is no lump-sum monetary transfer between consumers and platforms. We can similarly analyze the latter environments by, for example, assuming that consumers has some membership cost γ for each platform, and each platform can provide the stand-alone utility q_i by incurring cost $C(q_i)$. For ease of exposition. I stick to the case where the platforms can charge membership fees.

Examples The form of functions v and π differ depending on the specific design choice platform can make. For example, if the platform chooses transaction fees, v and π would be linear. However, if platforms' design choices include non-price instruments such as the investments in complements or introducing first-party products, the form of v and π may be nonlinear. I provide two examples below: transaction fees and first-party products.

Example 1. (transaction fees). Suppose that at each transaction, platform generates an efficiency gains δ_u and δ_v on consumer and seller sides, incurs per-transaction marginal cost $c > 0$, and charges per-transaction fee t_i to consumers. Then, writing u_i as $u_i = u_0 + \delta_u - t_i$, we can write v and π as $v(u_i) = v_0 + \delta_v$ and $\pi(u_i) = u_0 + \delta_u - c - u_i$.

Example 2. (Platform entry into product markets) Suppose that each seller has a unique product in a certain category, which has demand scale that follows $\theta \sim U[0, 1]$, which is the probability that a consumer purchases a product in that category. After observing the realization of demand scale θ, platform can choose whether to enter each category, incurring entry cost $\kappa > 0$. If platform does not enter, the seller derives per-consumer profit π^m. If platform enters, the seller and the platform derives per-consumer profit π^d and π^p, respectively. Assume that competition reduces producer surplus: $\pi^m > \pi^d + \pi^p$. Consumer surplus under monopoly and duopoly are given by u^m and u^d, respectively. Let $\hat{\theta}_i$ be the platform i's threshold value such that platform enters a category if and only if $\theta \geq \hat{\theta}_i$. Then, writing u_i as

$$u_i = u(\theta_i) := \int_{\theta_i}^{1} \theta u^d d\theta + \int_{0}^{\theta_i} \theta u^m d\theta = \frac{u^m + [1 - \theta_i^2](u^d - u^m)}{2},$$

we can write v and π as

$$v(u_i) = \int_{\theta(u_i)}^{1} \theta\pi^d d\theta + \int_{0}^{\theta(u_i)} \theta\pi^m d\theta = \frac{\pi^m - \{1 - [\theta(u_i)]^2\}(\pi^m - \pi^d)}{2},$$
$$\pi(u_i) = \int_{\theta(u_i)}^{1} (\theta\pi^p - \kappa) d\theta = \frac{\{1 - [\theta(u_i)]^2\}\pi^p}{2} - [1 - \theta(u_i)]\kappa,$$

where $\theta(u_i) = u^{-1}(u_i)$. In the direct channel, there is no platform entry but sellers incur fixed transaction costs ψ so that $u_0 = \pi^m/2$ and $v_0 = u^m/2 - \psi$.

4 Equilibrium analysis

4.1 Equilibrium under bundled usage

To analyze participation equilibria, I consider the consumers' incentives to join the platforms given a expectation that all sellers join all platforms. A consumer has an incentive to join platform i if

$$\Delta U_i(\mathbf{u}) \geq P_i,$$

where $\Delta U_i(\mathbf{u}) = U^{\mathcal{N}}(\mathbf{u}) - U^{\mathcal{N}\setminus\{i\}}(\mathbf{u})$ is the *incremental value* of platform i for consumers. Thus, the maximal membership fee P_i that guarantees the participation of consumers on platform i is given by $\Delta U_i(\mathbf{u})$. Given that all consumers join all the platforms, the probability that platform i is used by a consumer is given by

$$d_i^{\mathcal{N}}(\mathbf{u}) = \Pr\left(u_i + \epsilon_i = \max\left\{u_0, \max_{j\in\mathcal{N}}\{u_j + \epsilon_j\}\right\}\right)$$
$$= \int_{u_0 - u_i}^{\infty} f(\epsilon) \prod_{j\in\mathcal{N}\setminus\{i\}} F(\epsilon + u_i - u_j) d\epsilon.$$

Next, consider the incentives of sellers to join each platform, which depends on whether the usages are bundled or separate. Under the bundled usage, a seller has an incentive to join platform i if

$$\Delta V_i^B(\mathbf{u}) \geq T_i,$$

where $\Delta V_i^B(\mathbf{u}) = V^{\mathcal{N},\mathcal{N}}(\mathbf{u}) - V^{\mathcal{N},\mathcal{N}\setminus\{i\}}(\mathbf{u}) = d_i^{\mathcal{N}}(\mathbf{u})v(u_i)$ is the incremental value of platform i for sellers. Thus, the maximal membership fee charged to sellers is given by $T_i = d_i^{\mathcal{N}}(\mathbf{u})v(u_i)$.

Finally, I consider the first stage of the game where platforms choose policies $(u_i)_{i\in\mathcal{N}}$. Under the bundled usage, the optimal consumer utility u_i set by each platform is derived by solving the following maximization problem:

$$\max_{u_i} \Delta U_i(\mathbf{u}) + d_i^{\mathcal{N}}(\mathbf{u})[v(u_i) + \pi(u_i)] \tag{8}$$

By a simple calculation, we have $\partial \Delta U_i / \partial u_i = d_i^{\mathcal{N}}$. Then, the first-order condition with respect to u_i is given by

$$d_i^{\mathcal{N}} w'(u_i) + \frac{\partial d_i^{\mathcal{N}}(\mathbf{u})}{\partial u_i}[v(u_i) + \pi(u_i)] = 0. \tag{9}$$

Invoking the symmetry and, the condition (9) can be rewritten as

$$\chi^B(u^B, n) = w'(u^B) + \frac{v(u^B) + \pi(u^B)}{\Phi(u^B, n)} = 0, \tag{10}$$

where

$$\Phi^B(u, n) = \frac{d_i^{\mathcal{N}}(\mathbf{u}_n)}{\frac{\partial d_i(\mathbf{u}_n)}{\partial u_i}},$$

and $\mathbf{u}_n = (u, u, \ldots, u)$.

To guarantee that the above characterization is sufficient, I put the following assumption on the form of u, which states that the joint surplus of sellers and the platform is not too small so that platforms have an incentive to attract consumer participation.

Assumption 2. $v(u) + \pi(u) + \frac{f(u_0 - u)}{1 - F(u_0 - u)} > 0$ *for all* $u \in [\underline{u}, \overline{u}]$.

Then, we have the following characterization of the symmetric participation equilibrium.

Proposition 1. *In any participation equilibrium under bundled usage,* $T_i = \Delta V_i^B(\mathbf{u})$, *and* $P_i = \Delta U_i(\mathbf{u})$ *hold for any given* $\mathbf{u}$*. Further, given Assumptions 1 and 2, there is a unique symmetric participation equilibrium with* $u_i = u^B$ *that satisfies equation* (10).

Proof. In Appendix. □

4.2 Equilibrium under separate usage

Next, I analyze the equilibrium under separate usage. Given a set of platforms that each seller joins, consumers' usage probabilities are the same as that under bundled usage. Further, given an expectation over the set of platforms that sellers join, consumers' participation choices are the same as that under bundled usage. Thus, consumer choices under separate usage are virtually the same as that under bundled usage, given fixed seller participation.

What is different is sellers' incentive to join platforms. Under the separate usage, a seller has an incentive to join if

$$\Delta V_i^S(\mathbf{u}) \geq T_i,$$

where

$$\begin{aligned}\Delta V_i^S(\mathbf{u}) &= V^{\mathcal{N},\mathcal{N}}(\mathbf{u}) - V^{\mathcal{N}\setminus\{i\},\mathcal{N}\setminus\{i\}}(\mathbf{u}) \\ &= d_i^{\mathcal{N}}(\mathbf{u})[v(u_i) - v_0] - \sum_{j\in\mathcal{N}\setminus\{i\}} [d^{\mathcal{N}\setminus\{i\}}(\mathbf{u}) - d^{\mathcal{N}}(\mathbf{u})][v(u_j) - v_0]\end{aligned}$$

is the incremental value of platform i for sellers under separate usage.

As a result, the optimal consumer utility u_i set by each platform is derived by solving the following maximization problem:

$$\max_{u_i} \Delta U_i(\mathbf{u}) + \Delta V_i^S(\mathbf{u}) + d_i^{\mathcal{N}}(\mathbf{u})\pi(u_i). \tag{11}$$

The first-order condition with respect to u_i is given by

$$d_i^{\mathcal{N}} w'(u_i) + \frac{\partial d_i^{\mathcal{N}}(\mathbf{u})}{\partial u_i}\pi(u_i) + \sum_{j\in\mathcal{N}} \frac{\partial d_j^{\mathcal{N}}(\mathbf{u})}{\partial u_i}[v(u_j) - v_0] = 0. \tag{12}$$

Invoking the symmetry, the condition (12) can be rewritten as

$$\chi^S(u^S, n) = w'(u^S) + \frac{v(u^S) + \pi(u^S)}{\Phi(u^S, n)} - \Psi(u^S, n)v(u^S) - \Psi_0(u^S, n)v_0 = 0, \tag{13}$$

where

$$\Psi(u, n) = \frac{-\sum_{j\in\mathcal{N}\setminus\{i\}} \frac{\partial d_i(\mathbf{u}_n)}{\partial u_i}}{d_j^{\mathcal{N}}(\mathbf{u}_n)} > 0,$$

and

$$\Psi_0(u, n) = -\frac{\frac{\partial d_0^{\mathcal{N}}(\mathbf{u}_n)}{\partial u_i}}{d_i^{\mathcal{N}}(\mathbf{u}_n)} > 0.$$

To guarantee that the above characterization is sufficient, I put another assumption on the primitives, which states that the per-transaction seller surplus is higher on the platforms compared to that on direct channels.

Assumption 3. *For all $u \in [\underline{u}, \overline{u}]$, $v(u) \geq v_0$ holds.*

Given this assumption, the following characterization of equilibrium under separate usage is obtained.

Proposition 2. *Under Assumption 3, in any participation equilibrium under separate usage,* $T_i = \Delta V_i^S(\mathbf{u})$*, and* $P_i = \Delta U_i(\mathbf{u})$ *for any given* $\mathbf{u}$*. Further, given Assumptions 1 and 2, there exist a symmetric participation equilibrium with* $u_i = u^S$ *that satisfies equation* (13)*.*

Proof. In Appendix. $\square$

4.3 Impact of usage lock-in

Based on the analyses of equilibria under bundled usage and separate usage, I compare the values of equilibrium consumer utilities. The differences between equilibrium utilities are twofold. First, under separate usage, platforms take into account that the choice of consumer utilities affects the surplus sellers can obtain on other channels, whereas they do not under bundled usage. Because a higher level of consumer utility of one platform reduces the choice probabilities of other channels, platforms have incentives to set lower consumer utility under separate usage than under bundled usage. This observation leads to the following proposition.

Proposition 3. *The equilibrium consumer utility under bundled usage is higher than that under separate usage. That is,* $u^B > u^S$*.*

Proof. In Appendix. $\square$

This provides one implication of usage lock-in: usage lock-in shifts platform design in favor of consumers.

Below are the equilibrium consumer utilities under bundled usage for specific examples presented in the model section.

Example 1 (Continued)**.** In Example 1, $w'(u) = 0$ for all $u \in [\underline{u}, \overline{u}]$, so the equilibrium consumer utility is given by $v(u_i) + \pi(u_i) = v_0 + \delta_v + u_0 + \delta_u - c - u_i$, which implies that

$$u^B = v_0 + \delta_v + u_0 + \delta_u - c.$$

In terms of transaction fees, the equilibrium transaction fees t^B can be written as

$$t^B = c - \delta_v - v_0.$$

Example 2 (Continued)**.** In Example 2, rewrite the variables as

$$\begin{aligned}
\overline{ps} &= \pi^m \\
\overline{cs} &= u^d \\
\rho_u &= \frac{u^m}{u^d} \in (0,1) \\
\rho_\pi &= \frac{\pi^d + \pi^p}{\pi^m} \in (0,1).
\end{aligned}$$

Then, we have

$$w'(u_i) = \frac{\theta(u_i)[\overline{cs}(1-\rho_u) - \overline{ps}(1-\rho_\pi)) - \kappa}{\theta(u_i)\overline{cs}(1-\rho_u)},$$

and

$$v(u_i) + \pi(u_i) = \frac{\overline{ps}[\rho_\pi + \theta(u_i)^2(1-\rho_\pi)}{2} - (1-\theta(u_i))\kappa.$$

Suppose that $n = 1$ and $F(\epsilon) = 1 - \exp(-\lambda\epsilon)$. Then, the equilibrium condition for u^B can be written as

$$\left[1 - \frac{\overline{ps}(1-\rho_\pi)}{\overline{cs}(1-\rho_u)} - \frac{\kappa}{\theta(u^B)\overline{cs}(1-\rho_u)}\right]\frac{1}{\lambda} + \frac{\overline{ps}[\rho_\pi + \theta(u^B)^2(1-\rho_\pi)]}{2} - (1-\theta(u^B))\kappa = 0.$$

In this case, the likelihood of entry, $1-\theta(u^B)$ is increasing in $\overline{cs}$, decreasing in ρ_u, and increasing in ρ_π. $1-\theta(u^B)$ increases with $\overline{ps}$ if and only if $\kappa > \hat{\kappa}$ for some $\hat{\kappa}$. Further, $\hat{\kappa}$ is increasing in ρ_π.

4.4 Welfare properties of equilibrium

Given the equilibrium characterization, I show that equilibrium consumer utility is too high in terms of social welfare. This result arises from the incentives of platforms to divert the transactions from other channels.

The aggregate welfare when all consumers and sellers join all the platforms is given by

$$\begin{aligned}
W(\mathbf{u}) =& U^{\mathcal{N}}(\mathbf{u}) + d_0^{\mathcal{N}}(\mathbf{u})v_0 + \sum_{i\in\mathcal{N}} d_i^{\mathcal{N}}(\mathbf{u})[v(u_i) + \pi(u_i)] \\
=& U^{\mathcal{N}}(\mathbf{u}) + v_0 + \sum_{i\in\mathcal{N}} d_i^{\mathcal{N}}(\mathbf{u})[v(u_i) + \pi(u_i) - v_0],
\end{aligned} \tag{14}$$

which has the first derivative

$$\frac{\partial W(\mathbf{u})}{\partial u_i} = d_i w'(u_i) + \sum_{j\in\mathcal{N}} \frac{\partial d_j(u_B)}{\partial u_{Bi}}[v(u_j) + \pi(u_j) - v_0]. \tag{15}$$

Let u^O be the symmetric value of consumer utility that makes the above derivative zero.

Evaluating equation (15) at u^B, we have

$$\frac{\partial W(\mathbf{u}_n^B)}{\partial u_i} = -\sum_{j\in\mathcal{N}} \frac{\partial d_j^{\mathcal{N}}}{\partial u_i} v_0 + \sum_{j\in\mathcal{N}\backslash\{i\}} \frac{\partial d_j^{\mathcal{N}}}{\partial u_i} [v(u^B) + \pi(u^B)] < 0,$$

which implies that the equilibrium consumer utility under bundled usage u^B is too high. That is, equilibrium policies of platforms are too consumer friendly in terms of social welfare.

As a comparison, consider the welfare property of equilibrium consumer utility under separate usage. Evaluating equation (15) at u^S, we have

$$\frac{\partial W(\mathbf{u}_n^S)}{\partial u_i} = \sum_{j\in\mathcal{N}\backslash\{i\}} \frac{\partial d_j^{\mathcal{N}}}{\partial u_i} \pi(u^B).$$

This implies that the equilibrium consumer utility under separate usage is too high if and only if $\pi(u^B) > 0$ and $n > 1$.

The following proposition summarizes the results on the welfare properties of equilibrium consumer utilities.

Proposition 4. *The equilibrium consumer utility under bundled usage is always higher than the socially optimal level. The equilibrium consumer utility under separate usage is higher than the socially optimal level if and only if* $\pi(u^S) > 0$ *and* $n > 1$.

The fact that the equilibrium consumer utility under bundled usage is too high arises from a version of the business-stealing effect; each platform ignores the forgone profits of sellers and platforms when it attracts consumers by raising consumer utility. Because all the gain of consumers from increasing consumer utility is collected as an incremental value, each platform has an aligned incentive with consumers. Consequently, only the business-stealing effect distorts the equilibrium consumer utility, and excessive consumer utility result is obtained.

Below, I provide the comparison between equilibrium and socially optimal consumer utilities in specific examples provided before.

Example 1 (Continued)**.** In Example 1, the welfare-optimal consumer utility is given by

$$u^O = u_0 + \delta_u + \delta_v - c \tag{16}$$

In terms of transaction fees, the socially-optimal transaction fee t^O can be written as

$$t^O = c - \delta_v.$$

In Example 1, as the equilibrium transaction fees, the welfare-optimal transaction fees are below marginal cost by the size of efficiency gain δ_v on sellers' side. However, while the welfare-optimal transaction fees are below marginal cost only by its efficiency gain δ_v for sellers, the equilibrium transaction fees additionally have discounts that equal the baseline gains from trade v_0, which has nothing to do with the welfare gain from using the platform.

Example 2 (Continued)**.** Suppose that $n = 1$ and that $F(\epsilon) = 1 - \exp(-\lambda\epsilon)$ inn Example 2. Then u^O is given by the condition

$$\left[1 - \frac{\overline{ps}(1-\rho_\pi)}{\overline{cs}(1-\rho_u)} - \frac{\kappa}{\theta(u^O)\overline{cs}(1-\rho_u)}\right]\frac{1}{\lambda} - \overline{ps}(1-\rho_\pi)\left(\frac{1-\theta(u^O)^2)}{2}\right) - (1-\theta(u^O))\kappa + \psi = 0.$$

The likelihood of socially optimal platform entry $1 - \theta(u^O)$ is increasing in $\overline{cs}(1-\rho_u)$ but decreasing in $\overline{ps}(1-\rho_\pi)$.

The analysis of Example 2 shows that while socially-optimal platform entry into product markets decreases in the scale of producer surplus $\overline{ps}$, equilibrium platform entry into product markets may increase with the scale of producer surplus. This highlights the source of the divergence between the socially-optimal and equilibrium platform design. With a larger producer surplus, platforms have a strong incentive to divert consumers in order to increase the sellers' willingness to pay for platforms. However, this increase in the willingness to pay of sellers does not reflect the welfare, increasing the divergence between equilibrium and optimal level of entry.

5 Impact of platform competition under bundled usage

The fact that platforms have an incentive to set excessively high consumer utilities under bundled usage has rich implications on the impacts of platform competition on welfare. In the following, I focus on the case of bundled usage, which is the main interest of the analysis.

5.1 Platform entry

First, I consider the impact of platform entry captured by an increase in n. The next proposition shows that an entry of a platform increases the equilibrium consumer utilities.

Proposition 5. *If $w'(u^B) \neq 0$, u^B is increasing in n. If $w'(u^B) = 0$ for all $x \in [\underline{u}, \overline{u}]$, u^B is independent of n.*

In the rest of this subsection, I stick to a specification presented in Example 1 to highlight the key result that platform entry may lower the welfare.

In the setting of Example 1, the equilibrium consumer utility is given by $u^B = v_0 + \delta_v + u_0 + \delta_u - c$. Then, the consumer surplus is written as

$$U^{\mathcal{N}}(\mathbf{u}_n^B) = u_0 + \int_{u_B^0 - u^B}^{\infty} (\epsilon - u_0 + u^B) n f(\epsilon) F(\epsilon)^{n-1} d\epsilon \tag{17}$$

Thus, the equilibrium aggregate welfare is given by

$$\bar{W}(n) := u_0 + v_0 + \int_{c-\delta_u-\delta_v-v_0}^{\infty} (\epsilon + \delta_u + \delta_v - c) n f(\epsilon) F(\epsilon)^{n-1} d\epsilon \tag{18}$$

The first two terms are the surplus from trades that can be obtained through direct channels, and the third term is the additional surplus generated by the trade on the platforms. However, because the transaction prices are inefficiently low, the transaction through the platform generates not only the surplus, but also the cost, which is the case when $\epsilon \in [c - \delta_u - \delta_v - v_0, c - \delta_u - \delta_v]$.

Whenever $\underline{\epsilon} < c - \delta_u + \delta_v + v_0$ holds, there are some inefficient transactions on platforms. In some extreme cases where u_S^0 is sufficiently large, abandoning platforms may improve the welfare. To see this, note that the welfare impact of the existence of the platforms is given by

$$\bar{W}(n) - \bar{W}(0) = \int_{c-\delta_u-\delta_v-v_0}^{\infty} (\epsilon + \delta_u + \delta_v - c) n f(\epsilon) F(\epsilon)^{n-1} d\epsilon$$

Since the region where inefficient transactions take place becomes broader as v_0 grows, the presence of platforms may lower the welfare when v_0 is sufficiently large.

Next, I analyze how welfare varies with the number of platforms. An increase in the number of platforms affects welfare in two ways. Firstly, as the number of options for consumers increases, the probability of an inefficient transaction decreases, which improves welfare. Second, as the number of platforms increase, the number of the transaction itself increases, which may increase the number of inefficient transactions and decrease welfare. When the former dominates the latter, welfare improves as the number of platforms increases and vice versa. Proposition 5 states that the welfare-improving effects dominate if and only if the number of platforms is greater than some critical value.

The following proposition formalizes the above discussion.

Proposition 6. *Suppose that* $\int_{\underline{\epsilon}}^{\infty} (\epsilon) n f(\epsilon) F(\epsilon)^{n-1} d\epsilon + \delta_u + \delta_v < c$. *Then,*

1. *for any fixed* n*, there exists* $\hat{v}_0 > 0$ *such that* $\bar{W}(n) - \bar{W}(0) < 0$ *if and only if* $v_0 > \hat{v}_0$*, and*

2. $\bar{W}(n)$ *is U-shaped. That is, there exists* $\hat{n} \geq 0$ *such that* $\bar{W}(n)$ *increases with* n *for all* $n > \hat{n}$*, and decreases with* n *for all* $n < \hat{n}$

Proposition 6 suggests that when platforms' transaction fees are so low that even the presence of the platforms hurt welfare, fostering platform competition may not mitigate the problem unless the competition is already intensive.

5.2 Platform merger

As observed in the analysis of entry, intensifying competition may have an adverse effect on welfare. On the flip side, a reduction in competition may improve welfare. As a stark example, I show an analysis of the impact of mergers on welfare.

For ease of exposition, consider a merger to monopoly, labeled as M. I assume that the merger leads to an integration of platforms, in which case consumers' benefit from usage ϵ^M is given by

$$\epsilon^M = \max_{j \in \mathcal{N}} \epsilon_j.$$

This formulation of mergers is adopted in the context of mergers in auction environments, as Loertscher and Marx (2019).

Then, the merged entity chooses consumer utilities to maximize the joint profit

$$\begin{aligned}
&U^{\{M\}}(u_M) - U^{\emptyset}(u_M) + d_M^{\{M\}}(u_M)[v(u_M) + \pi(u_M)] \\
=& \int_{u_0 - u_M}^{\infty} [\epsilon - u_0 + u_M] n f(\epsilon) F(\epsilon)^{n-1} d\epsilon + [1 - F(u_0 - u_M)^n][v(u_M) + \pi(u_M)]
\end{aligned}$$

which is maximized by the first-order condition

$$\Phi^M(u_M, n) w'(u_M) + v(u_M) + \pi(u_M) = 0, \tag{19}$$

where $\Phi^M(u_M, n) = d_M^{\{M\}}(u_M) / \left(\frac{\partial d_M^{\{M\}}(u_M)}{\partial u_M} \right)$, and $d_M^{\{M\}}(u) = 1 - F(u)^n$. Let u^M be the equilibrium consumer utility set by the monopoly. The following proposition shows that the merger lowers the consumer utility, but the consumer utility is still excessively high, implying that the merger to monopoly always improves welfare.

Proposition 7. *Consider a merger to monopoly. Let u^O be the pre-merger first-best policy, u^B the pre-merger symmetric equilibrium policy, and u^M the post-merger equilibrium policy. Then, $u^O < u^M \leq u^B$. If $u^M \neq u^B$, then the merger raises welfare.*

Proposition 7 states that a merger to monopoly improves welfare. This is a sharp contrast with the standard welfare-reducing effects of mergers.

5.3 Limit pricing

Amelio, Giardino-Karlinger and Valletti (2020) used a model of platform competition with singlehoming agents to examine the incentive of an incumbent platform to engage in limit pricing when an incumbent platform can set the price before the entry of a potential entrant. Using a specification of Example 1, I examine how the incumbent's incentive to engage in limit pricing and its welfare implications. In particular, the presence of entry pressure may lower even the short-run welfare when the incumbent engages in limit pricing, because the monopoly consumer utility is already too high in terms of social welfare.

Consider the following extension of the baseline model. There is an incumbent platform I and a potential entrant platform E that needs to incur entry cost K to enter. The timing is given as follows:

1. The incumbent platform I first set consumer utility u_I.

2. Observing the consumer utility set by I, the entrant platform E chooses whether to enter and the consumer utility u_E if it enters.

The remaining timing is the same as that of the main model.

To derive the equilibria, we first need to compute the profit of platform E when it enters, given the consumer utility u_I set by the incumbent. A calculation analogous to that of main analysis shows that platform E sets $u_E = u^B = \delta_u + u_0 + \delta_v + v_0 - c$. Thus, the probability that the platform E is used by consumers is given by

$$\begin{aligned} d_E(u_I, u^B) &= \int_{u_0-u^B}^{\infty} f(\epsilon)F(\epsilon - u_I + u^B)d\epsilon \\ &= \int_{u_0-u_I}^{\infty} f(\epsilon + u_I - u^B)F(\epsilon)d\epsilon. \end{aligned}$$

Post-entry consumer surplus is given by

$$U^{\{I,E\}}(u_I, u^B) = u_0 + \int_{u_0-u_I}^{\infty} [1 - F(\epsilon)F(\epsilon + u_I - u^B)]d\epsilon,$$

and consumers' surplus when they join only platform I is given by

$$U^{\{I\}}(u_I, u^B) = u_B + \int_{u_0-u_I}^{\infty} [1 - F(\epsilon)]d\epsilon.$$

Thus, the incremental value of platform E is given by

$$U^{\{I,E\}}(u_I, u^B) - U^{\{I\}}(u_I, u^B) = \int_{u_0-u_I}^{\infty} F(\epsilon)[1 - F(\epsilon + u_I - u^B)]d\epsilon.$$

Finally, the profit of the platform E when it commits to consumer utility u_I is given by

$$\Pi_E(u_I) = \int_{u_0-u_I}^{\infty} F(\epsilon)[1 - F(\epsilon + u_I - u^B)]d\epsilon - K.$$

Given u_I, platform E enters if and only if $\Pi_E(u_I) > 0$. Consequently, platform I has three alternative strategies in choosing u_I:

1. Blockade: choose $u_I = u^B$, and $\Pi_E(u^B) \leq 0$ holds. As a result, platform E does not enter.

2. Deter: choose u_I so that $\Pi_E(u_I) \leq 0$. As a result, platform E does not enter.

3. Accommodate: choose $tb_I = u^B$, and $\Pi_E(u^B) > 0$ holds. As a result, platform E enters.

I consider the case where $\Pi_E(u^B) > 0$ holds, that is, the blockaded entry does not take place. Let $\bar{K}$ be the entry cost such that $\Pi_E(u^B) = 0$. Then, $\Pi_E(u^B) > 0$ holds if and only if $K < \bar{K}$. Given this assumption, platform I chooses between deterring and accommodating the entry. The profit of platform I from accommodating the entry is given by

$$\Pi_I^a = \int_{u^B}^{\infty} F(\epsilon)[1 - F(\epsilon)]d\epsilon.$$

Next, consider the profit of platform I when it deters the entry. At the consumer utility u_I that deters entry, the profit of platform I is given by

$$\Pi_I^d(u_I) = \int_{u_0 - u_I}^{\infty} [1 - F(\epsilon)]d\epsilon + [1 - F(u_0 - u_I)](u_0 + \delta_u + v_0 + \delta_v - c - u_i),$$

subject to $\Pi_E(u_I) \leq 0$. Then, there exists $\hat{u} > u^B$ such that $\Pi_I^d(\hat{u}) = \Pi_I^a$, that is, the consumer utility that makes the profit of the incumbent in the case of entry deterrence equal to the profit in the case of accommodating the entry. Further, define $\tilde{u}(K)$ by $\Pi_E(\tilde{u}(K)) = 0$, that is, the minimal consumer utility that the incumbent needs to set in order to deter the entry. Then, if $\tilde{u}(K) < \hat{u}$, platform I can profitably deter the entry by setting $u_I = \tilde{u}(K)$. It can be shown that there exists $\hat{K} > 0$ such that $\tilde{u}(K) = \hat{u}$, and $\Pi_I^d(\tilde{u}(K)) > \Pi_I^a$ for all $K > \hat{K}$. Consequently, the following result is obtained.

Proposition 8. *For $K \in (0, \hat{K})$, platform I accommodates the entry. For $K \in [\hat{K}, \bar{K})$, platform I deters the entry by setting $u_I = \tilde{u}(K) > u^B$, and the welfare is lower than when there is no potential entrant.*

Proof. In Appendix. □

When the value of entry cost is intermediate, platform I engages in limit pricing by setting consumer utility above the level that would be set without a threat of entry. Because the consumer utilities without the threat of entry are already too high, further raising consumer utilities by limit pricing always lowers the welfare. This result is in contrast with the standard welfare effects of limit pricing in ordinary markets, where the reduction in prices improves welfare, at least in the short-run. In summary, the adverse effects of limit pricing may be serious in two-sided markets than in ordinary markets.

6 Extension and Discussion

In this section, I discuss how the welfare implications of equilibrium would change if the framework of this study is extended.

Mixed homing Suppose that consumers differ in stand-alone benefits for platforms and choose whether to singlehome or multihome. Consider the following duopoly setting where

$$u^{\mathcal{P}}(u_1, u_2) = \begin{cases} U^{\{1\}}(u_1, u_2) - P_1 - \tau x & \text{if } \mathcal{P} = \{1\} \\ U^{\{2\}}(u_1, u_2) - P_2 - \tau(1-x) & \text{if } \mathcal{P} = \{2\} \\ U^{\{1,2\}}(u_1, u_2) - P_1 - P_2 - \tau - \Delta & \text{if } \mathcal{P} = \{1, 2\}, \end{cases}$$

where $\Delta > 0$ is a multihoming cost. In addition assume that there is no direct channel, which amounts to assuming that $u_0 = -\infty$.

Then defining the critical types of consumers as

$$\hat{x}_1 = \frac{U^{1,2} - U^{\{2\}} - P_1 - \Delta}{\tau}$$

and

$$\hat{x}_2 = 1 - \frac{U^{1,2} - U^{\{1\}} - P_2 - \Delta}{\tau},$$

we have that consumers in $[0, \hat{x}_2]$ join only platform 1, consumers in $[\hat{x}_2, \hat{x}_1]$ joint both platforms, and consumers in $[\hat{x}_1, 1]$ join only platform 2. I slightly change the timing in the way that platforms simultaneously set consumer membership fees and consumer utilities. After observing these actions, platforms choose seller membership fees.

Note that because there is no choice but choosing platform i when a consumer singlehomes platform i, $\mathcal{U}^{\{i\}}$ is given by

$$U^{\{i\}} = u_i + \mathbb{E}[\epsilon_i].$$

Then, platform 1's profit is given by

$$\hat{x}_1 P_1 + \left\{ \hat{x}_1 d_1^{\{1,2\}} + \hat{x}_2 \left[1 - d_1^{\{1,2\}} \right] \right\} [v(u_1) + \pi(u_1)].$$

Noting that

$$\frac{\partial \hat{x}_1}{\partial P_1} = -\frac{1}{\tau},$$
$$\frac{\partial \hat{x}_1}{\partial u_1} = \frac{d_1^{\{1,2\}}}{\tau}$$
$$\frac{\partial \hat{x}_2}{\partial u_1} = \frac{1-d_1^{\{1,2\}}}{\tau},$$

the first-order condition for platform 1's optimal membership fee P_1 for consumers is

$$\hat{x}_1 + \frac{\partial \hat{x}_1}{\partial P_1}\left\{P_1 + d_1^{\{1,2\}}[v(u_1)+\pi(u_1)]\right\} = 0$$

Note that this implies that

$$\hat{x}_1 d^{\{1\}} - \frac{\partial \hat{x}_1}{\partial u_1}\left\{P_1 + d_1^{\{1,2\}}[v(u_1)+\pi(u_1)]\right\} = 0.$$

Then, the first-order condition for usage level is given by

$$\left\{\hat{x}_1 d_1^{\{1,2\}} + \hat{x}_2\left[d_1^{\{1\}} - d_1^{\{1,2\}}\right]\right\}[v'(u_1)+\pi'(u_1)] + \hat{x}_1 d_1^{\{1,2\}}$$
$$+\left\{(\hat{x}_1 - \hat{x}_2)\frac{\partial d_1^{\{1,2\}}}{\partial u_1} + \frac{[1-d_1^{\{1,2\}}]^2}{\tau}\right\}[v(u_1)+\pi(u_1)].$$

Combining these two first-order conditions, the symmetric equilibrium consumer utility is given by

$$\hat{x}_1 w'(u) + \hat{x}_2[v'(u)+\pi'(u)] + \left\{(\hat{x}_1-\hat{x}_2)2\int_{\underline{\epsilon}}^{\infty} f^2(\epsilon)\epsilon + \frac{1}{2\tau}\right\}[v(u)+\pi(u)]. \tag{20}$$

Let $\hat{u}(\Delta)$ be the symmetric-equilibrium level of consumer utility. Then, we obtain the following result on the relation between consumer homing and equilibrium usage competition.

Proposition 9. *$\hat{u}(\Delta)$ is decreasing in Δ.*

Proof. In Appendix. □

This proposition implies that multihoming cost lowers the equilibrium consumer utilities, which implies that consumer multihoming is a key driving force that leads to an excessively high equilibrium consumer utility.

Seller heterogeneity I also have assumed that there is no ex-ante heterogeneity in consumers and sellers. This is not innocuous in terms of the welfare properties of equilibrium prices because when there is an ex-ante heterogeneity in usage benefit or membership utility, the prices that platforms set tend to be too high (Weyl, 2010). To incorporate this feature, I introduce the heterogeneity in the gains from trade of sellers obtained through the transaction in platforms. Consider the case of monopoly and suppose that $\delta_v \sim G$ with continuous density function g. Then, platforms set (u_M, P_M, T_M) to maximize their own profits. Then, the number of sellers who join platform i is given by $n_M^S = 1 - G\left(v_0 + \frac{T_M}{d_M^{\{M\}}}\right)$. For an analytical ease, I treat n_i^S as the choice variable of platform i, because it can be achieved by setting $T_M = [G^{-1}(1 - n_M^S) + v_0]n_M^S$. The profit of the platform at participation equilibrium is given by

$$U_M^{\{M\}} - U^{\emptyset} + n_i^S d_i(u_0 + \delta_u + v_0 + \delta_M - u_M - c),$$

where

$$\delta_M = v_0 + \frac{T_M}{d_M^{\{M\}}}.$$

At symmetric equilibrium, $\partial U^{\{M\}}/\partial n_i^S = \int_{\frac{u_0}{n_M^S} - u_M}^{\infty} (\epsilon + u_M) f(\epsilon) F(\epsilon)^{n-1} d\epsilon$ holds. Then, taking the first-order conditions with respect to n_M^S and u_M, we obtain

$$u_M = u_0 + \delta_u + v_0 + \delta_M - c,$$

and

$$\delta_M = \frac{1 - G(\delta_M)}{g(\delta_M)} + c - \bar{\epsilon}\left(\frac{u_0}{n_M^S} - u_M\right) - u_0 - v_0 - \delta_u,$$

where

$$\bar{\epsilon}(a) = \frac{\int_a^{\infty} \epsilon f(\epsilon) F(\epsilon)^{n-1} d\epsilon}{\int_a^{\infty} f(\epsilon) F(\epsilon)^{n-1} d\epsilon}$$

is the average gains from using a platform conditional on using the platform.

Next, consider the welfare properties of equilibrium consumer utilities. The aggregate surplus is given by

$$W = U^{\{M\}} + n_M^S d_M^{\{M\}}(u_0 + \delta_u + v_0 + \bar{\delta}(\delta_v) - c),$$

where

$$\bar{\delta}(b) = \frac{\int_b^{\infty} \delta g(\delta) d\delta}{1 - G(b)}$$

is the average additional gains from trade of sellers obtained through a platform.

Evaluating the derivatives of W at the equilibrium consumer utilities, we obtain

$$\left.\frac{\partial W}{\partial n_M^S}\right|_{n_M^S=\hat{n}_M^S, u_M=\hat{u}_M} = \underbrace{d_M^{\{M\}}\frac{1-G(\hat{\delta}_M)}{g(\hat{\delta}_M)}}_{\text{market power distortion}} > 0$$

$$\left.\frac{\partial W}{\partial u_M}\right|_{n_M^S=\hat{n}_M^S, u_M=\hat{u}_M} = \frac{\partial d_M^{\{M\}}}{\partial u_M}\hat{n}_M^S\left[\underbrace{\bar{\delta}(\hat{\delta}_M)-\hat{\delta}_M}_{\text{Spence distortion}} - \underbrace{\frac{1-G(\hat{\delta}_M)}{g(\hat{\delta}_M)}}_{\text{inefficient discount}}\right].$$

As described by Weyl (2010), when there is ex-ante heterogeneity in usage benefits, Spence distortion tends to lead the equilibrium price to be higher than the socially optimal price level. This factor mitigates and may overturn the welfare distortion derived in the main analysis. In total, whether the equilibrium consumer utilities are too high or too low depends on the relative size of Spence distortion. For example, when $g(\delta) = \lambda e^{-\lambda\delta}$, we have

$$\bar{\delta}(\delta) - \delta - \frac{1-G(\delta)}{g(\delta)} = -\left(1-e^{-\lambda\delta}\right)\left(\delta+\frac{1}{\lambda}\right) < 0.$$

Thus, in a specific example of exponentially distributed seller surplus, inefficiently high discounts dominates the Spence distortion, and excessive consumer utility results. By contrast, when $\delta \sim U[0,\theta]$, we have

$$\bar{(\delta)} - \delta - \frac{1-G(\delta)}{g(\delta)} = \frac{(\theta-\delta)(\theta-2)}{2\theta},$$

which is positive if and only if $\theta > 2$. Thus, in the case where $\theta > 2$, Spence distortion dominates, and insufficient consumer utility results.

In total, the overall welfare property of equilibrium consumer utility depends on the relative sizes of diversion incentives and Spence distortions, and both excessive and insufficient consumer utilities arise under natural conditions.

7 Conclusion

In this study, I have examined platform competition when consumers and sellers multi-home and platform compete for the usage with each other and direct channels. Platforms have incentives to set too consumer utilities. Such excessively high consumer utility may render the existence of platforms detrimental to welfare. In this regard, naive promotion of competition, such as promoting the platform's entry, may not improve the welfare depending on the initial market structure. Alternatively, the policy that alters usage mode from bundled usage to separate usage by lowering switching costs may be a better alternative.

There are several avenues for future research. First, allowing for an arbitrary heterogeneity in membership utilities generate the mixture of users that use different portfolios of platforms. However, characterizing users' optimal choice of the combination of the platforms to join is in general intractable when the number of platforms exceeds two. Finding a nice way to characterize users' membership decisions would be interesting future research. Second, bridging the bundled usage and separate usage by introducing the economies of scope for usage enables us to analyze the condition under which the bundled usage or the separate usage is likely, which is also an interesting avenue for future research.

Appendix

Proof of Proposition 1 Consider any subgame given a profile x of policies. I first show that consumers' gross utility from joining platforms is "quasi-concave" in the following sense: For any $\mathcal{P}$ and $\mathcal{T} \subset \mathcal{P}$ such that $i \in \mathcal{T}$, the following inequality

$$U^{\mathcal{P}}(\mathbf{u}) - U^{\mathcal{P}\setminus\{i\}}(\mathbf{u}) \leq U^{\mathcal{T}}(\mathbf{u}) - U^{\mathcal{T}\setminus\{i\}}(\mathbf{u}) \tag{21}$$

holds.

$$\begin{aligned} U^{\mathcal{P}}(\mathbf{u}) &= u_0 + \sum_{k\in\mathcal{P}} \int_{u_0-u_k}^{\infty} (\epsilon - u_0 + u_k) f(\epsilon) \Pi_{l\neq k} F(\epsilon - u_{Bl} + u_{Bk}) d\epsilon \\ &= u_0 + \int_{u_0-u_i}^{\infty} (\epsilon - u_0 + u_i) \sum_{k\in\mathcal{P}} f(\epsilon - u_k + u_i) \prod_{l\neq j} F(\epsilon - u_l + u_i) d\epsilon \\ &= u_0 + \int_{u_0-u_i}^{\infty} \left[1 - \prod_{k\in\mathcal{P}} F(\epsilon - u_k + u_i)\right] d\epsilon. \end{aligned} \tag{22}$$

The equality in the second line follows from change of variables from ϵ to $\epsilon - u_k + u_i$ for each $k \in \mathcal{P}$, and the equality in the third line follows from integral by parts.

Thus, we have

$$U^{\mathcal{P}}(\mathbf{u}) - U^{\mathcal{P}\setminus\{i\}}(\mathbf{u}) = \int_{u_B^0 - u_{Bi}}^{\infty} \prod_{k\in\mathcal{P}\setminus\{i\}} F(\epsilon - u_k + u_i)[1 - F(\epsilon)] d\epsilon.$$

Applying this equation, we have

$$\begin{aligned}
&U^{\mathcal{P}}(\mathbf{u}) - U^{\mathcal{P}\setminus\{i\}}(\mathbf{u}) - [U^{\mathcal{T}}(\mathbf{u}) - U^{\mathcal{T}\setminus\{i\}}(\mathbf{u})] \\
=& -\int_{u_0-u_i}^{\infty} \prod_{k\in\mathcal{T}\setminus\{i\}} F(\epsilon - u_k + u_i)\left[1 - \prod_{j\in\mathcal{P}\setminus\mathcal{T}} F(\epsilon - u_j + u_i)\right][1-F(\epsilon)]d\epsilon < 0,
\end{aligned}$$

which shows that the condition (21) holds.

Next, I show that in any equilibrium, consumers and sellers join all the platforms. Suppose that consumers join the set $\mathcal{P} \subset \mathcal{N}$ of platforms and let $i \notin \mathcal{P}$. In this case, platform i earns 0 profit. By setting $P_i = \hat{P}_i = U^{\mathcal{P}\cup\{i\}}(\mathbf{u}) - U^{\mathcal{P}}(\mathbf{u})$ and $T_i = \hat{T}_i = d_i u_S(x_i)$, consumers choose the set $\mathcal{P}'$ such that $i \in \mathcal{P}'$ because

$$\begin{aligned}
\max_{\mathcal{T}\subset\mathcal{N}\setminus\{i\}} \left[U^{\mathcal{T}}(\mathbf{u}) - \sum_{j\in\mathcal{T}} P_j\right] &= U^{\mathcal{P}}(\mathbf{u}) - \sum_{j\in\mathcal{P}} P_j \\
&\leq U^{\mathcal{P}\cup\{i\}}(\mathbf{u}) - \sum_{j\in\mathcal{P}} P_j - \hat{P}_i \\
&\leq \max_{\mathcal{T}\subset\mathcal{N}\setminus\{i\}} \left[U^{\mathcal{T}\cup\{i\}}(\mathbf{u}) - \sum_{j\in\mathcal{T}} P_j - \hat{P}_i\right].
\end{aligned}$$

Thus, the deviation to $(\hat{P}_i, \hat{T}_i)$ gives the profit

$$\begin{aligned}
&\int_{u_0-u_i}^{\infty} \prod_{k\in\mathcal{P}} F(\epsilon - u_k + u_i)[1-F(\epsilon)]d\epsilon + [v(u_i) + \pi(u_i)]\int_{u_0-u_i}^{\infty} f(\epsilon)\prod_{k\in\mathcal{P}} F(\epsilon - u_k + u_i)d\epsilon \\
=& \int_{u_0-u_i}^{\infty} \prod_{k\in\mathcal{P}} F(\epsilon - u_k + u_i)[1-F(\epsilon)]\left[v(u_i) + \pi(u_i) + \frac{f(\epsilon)}{1-F(\epsilon)}\right]d\epsilon
\end{aligned}$$

is profitable for platform i by Assumption 2. Therefore, in the equilibrium all consumers join all the platforms. Given that all consumers join all the platforms, the maximal membership fee each platform can charge is given by

$$P_i = U^{\mathcal{N}}(\mathbf{u}) - U^{\mathcal{N}\setminus\{i\}}(\mathbf{u})$$

for all $i \in \mathcal{N}$. Thus, the profile of membership fees $(P_j)_{j\in\mathcal{N}}$ that satisfies the above condition is the equilibrium membership fees for consumers.

Similarly, sellers join all the platforms. Suppose that platform i sets the membership fee T_i on the seller side such that sellers do not join platform i. Then, the platform earns zero profit because its incremental value is 0. Deviating to $T_i = 0$ attracts sellers to join and increases the incremental value of platform i to consumers, which allows platforms to increase P_i, increasing the profit of platform i. Thus platforms set membership fees that induce all sellers to join. The maximal price that the platform can charge to sellers is given by $T_i = d_i^{\mathcal{N}}(\mathbf{u})v(u_i)$ in the case of bundled usage.

Finally, I show that there is a unique symmetric participation equilibrium. To show this, I show that the equation (10) has the unique solution. By Assumption 1 we have $\chi(\underline{u}, n) > 0$ and $\chi(\overline{u}, n) < 0$. Thus, showing that

$$\begin{aligned}&\frac{\partial \chi^B(u^B, n)}{\partial u}\\ =&w''(u^B) + \frac{\frac{\partial \Phi(u^B,n)}{\partial u}}{\Phi(u^B, n)^2}[v'(u^B) + \pi'(u^B)] + \frac{v''(u^B) + \pi''(u^B)}{\Phi(u^B, n)} < 0\end{aligned} \tag{23}$$

establishes that $\chi(u, n) = 0$ has a unique solution in $(\underline{u}, \overline{u})$. Lemma 4 of Zhou (2017) implies that $\partial\Phi/\partial u > 0$. Further we have $w'(u^B) \leq 0$ and $v(u^B)+\pi(u^B) \geq 0$. To see this, suppose to the contrary that $w'(u^B) > 0$ or $v(u^B)+\pi(u^B) < 0$, then equation (10) implies that $w'(u^B) > 0$ and $v(u^B)+\pi(u^B) < 0$. However, Assumption 1 implies that w is concave and $w'(u^B) \leq 0$, a contradiction. Thus, we must have $w'(u^B) \leq 0$ and $v(u^B)+\pi(u^B) \geq 0$. Finally, we have $v'(u^B) + \pi'(u^B) < 0$ because $w'(u^B) \leq 0$. These imply that all terms in equation (23) are non-positive and some are negative. This establishes that the solution to equation (10) is unique. □

Proof of Proposition 2 The equilibrium pricing given the profile of consumer utilities $\mathbf{u}$ is the same as that under bundled usage except for the incremental values for sellers, which is given by

$$\Delta^{\mathcal{P}} V_i(\mathbf{u}) = V^{\mathcal{P},\mathcal{P}}(\mathbf{u}) - V^{\mathcal{P}\setminus\{i\},\mathcal{P}\setminus\{i\}}(\mathbf{u}) = d_i^{\mathcal{P}}(\mathbf{u})[v(u_i)-v_0] - \sum_{j\in\mathcal{P}\setminus\{i\}} [d^{\mathcal{P}\setminus\{i\}}(\mathbf{u}) - d^{\mathcal{P}}(\mathbf{u})][v(u_j)-v_0].$$

I show that $\Delta^{\mathcal{P}} V_i(\mathbf{u}) \leq \Delta^{\mathcal{T}} V_i(\mathbf{u})$ for all $i \in \mathcal{T} \subset \mathcal{P} \subset \mathcal{N}$, which establishes the sufficiency of the seller membership fees $T_i = \Delta V_i^S(\mathbf{u})$ for the participation equilibrium. This is shown by observing that

$$\begin{aligned}\Delta^{\mathcal{P}} V_i(\mathbf{u}) - \Delta^{\mathcal{T}} V_i(\mathbf{u}) =& \sum_{j\in\mathcal{P}} d_j^{\mathcal{P}}(v_j - v_0) - \sum_{j\in\mathcal{T}} d_j^{\mathcal{T}}(v_j - v_0)\\ &- \sum_{j\in\mathcal{P}\setminus\{i\}} d_j^{\mathcal{P}\setminus\{i\}}(v_j - v_0) + \sum_{j\in\mathcal{T}\setminus\{i\}} d_j^{\mathcal{T}\setminus\{i\}}(v_j - v_0)\\ =& -[d_i^{\mathcal{T}} - d_i^{\mathcal{P}}](v_i - v_0)\\ &- \sum_{j\in\mathcal{P}\cap\mathcal{T}\setminus\{i\}} [d_j^{\mathcal{T}} - d_j^{\mathcal{T}\setminus\{i\}} - d_j^{\mathcal{P}} + d_j^{\mathcal{P}\setminus\{i\}}](v_j - v_0)\\ &- \sum_{j\in\mathcal{P}\setminus\mathcal{T}} [d_j^{\mathcal{P}\setminus\{i\}} - d_j^{\mathcal{P}}](v_j - v_0)\\ \leq& 0\end{aligned}$$

which follows from the assumption that $v_j \geq v_0$ by Assumption 3 and the facts that $d_i^{\mathcal{P}'} \geq d_i^{\mathcal{P}}$ for any $\mathcal{P}' \subset \mathcal{P}$, and that

$$
\begin{aligned}
& d_j^{\mathcal{T}} - d_j^{\mathcal{T}\setminus\{i\}} - d_j^{\mathcal{P}} + d_j^{\mathcal{P}\setminus\{i\}} \\
= & \int_{u_0-u_j}^{\infty} f(\epsilon) \prod_{k\in\mathcal{P}\cap\mathcal{T}\setminus\{j,i\}} F(\epsilon - u_k + u_j) \left[1 - \prod_{k\in\mathcal{P}\setminus\mathcal{T}} F(\epsilon - u_k + u_j)\right] [1 - F(\epsilon - u_i + u_j)] d\epsilon > 0
\end{aligned}
$$

Thus, in a same way as the proof of Proposition 1, the equilibrium membership fees for sellers can be characterized by the incremental value $T_i = \Delta V_i^S(\mathbf{u})$.

As a result, each platform's choice of consumer utility u_i is given by (11). Then, the first-order condition for the symmetric equilibrium consumer utility is given by equation (13).

Proof of Proposition 3 Pick any value u^S that satisfy equation (13). Evaluating $\chi^B(u^S, n)$, we have

$$\chi^B(u^S, n) = \Psi(u^S, n)v(u^S) + \Psi_0(u^S, n)v_0 > 0.$$

Then, $\chi^B(u^S, n) > 0$ implies that $u^S < u^B$.

Proof of Proposition 5 By equation 10, if $w'(u) = 0$ for all $u \in [\underline{u}, \overline{u}]$, u^B is uniquely determined by $v(u^B)+\pi(u^B) = 0$. Thus, u^B is independent of n. Noting that Zhou (2017) shows that $\Phi(u, n)$ is decreasing in n, u^B is increasing in n if $w'(u^B) \neq 0$.

Proof of Proposition 6 The fact that $\bar{W}(n) - \bar{W}(0)$ is decreasing in v_0, and that $\max_{v_0\in[0,\underline{\epsilon}-c+\delta_u+\delta_v]}\{\bar{W}(n) - \bar{W}(0)\} = \int_{c-\delta_u+\delta_v}^{\infty}(\epsilon - c + \delta_u + \delta_v)nf(\epsilon)F(\epsilon)^{n-1}d\epsilon$, and that $\min_{v_0\in[0,\underline{\epsilon}-c+\delta_u+\delta_v]}\{\bar{W}(n) - \bar{W}(0)\} = \int_{\underline{\epsilon}}^{\infty}(\epsilon)nf(\epsilon)F(\epsilon)^{n-1}d\epsilon + \delta_u + \delta_v - c$ immediately implies the result.

Next, I show that $\bar{W}(n)$ is U-shaped. By integral by parts and some manipulations, we have

$$\bar{W}(n) = \left[u_0 + F(c - \delta_u - \delta_v - v_0)^n v_0 + \int_{c-\delta_u-\delta_v-v_0}^{\infty} [1 - F(\epsilon)^n] d\epsilon\right].$$

Then, we have

$$\bar{W}'(n) = \left[\log[F(c - \delta_u - \delta_v - v_0)]F(c - \delta_u - \delta_v - v_0)^n v_0 - \int_{c-\delta_u-\delta_v-v_0}^{\infty} \log[F(\epsilon)]F(\epsilon)^n d\epsilon\right]$$

Suppose that there exists $\hat{n}$ such that $\bar{W}'(\hat{n}) = 0$. Then, we have

$$\bar{W}''(\hat{n}) = \int_{c-\delta_u-\delta_v-v_0}^{\infty} \log[F(\epsilon)]\{\log[F(c-\delta_u-\delta_v-v_0)] - \log[F(\epsilon)]\}F(\epsilon)^n d\epsilon > 0.$$

Thus, if there exists $\hat{n}$ such that $\bar{W}'(\hat{n}) = 0$, $\bar{W}'(n) > 0$ for all $n > \hat{n}$ and $\bar{W}'(n) < 0$ for all $n < \hat{n}$.

Proof of Proposition 7 Note that if all policies are symmetric and given by x, the expression for the social welfare is the same before and after the merger. Then, showing that $u^O < u^M \leq u^B$ establishes the proposition. To show that $u^M \leq u^B$, by comparison between equations (10) and (19), it suffices to show that $\Phi(u, n) < \Phi^M(u, n)$, which is shown by the following relations:

$$\begin{aligned}
\Phi(u,n) &= \frac{\int_{u_0-u}^{\infty} f(\epsilon)F(\epsilon)^{n-1}d\epsilon}{f(u_0-u)F(u_0-u)^{n-1} + \int_{u_0-u}^{\infty} f(\epsilon)^2 F(\epsilon)^{n-2}d\epsilon} \\
&= \frac{1 - F(u_0-u)^n}{nf(u_0-u)F(u_0-u)^{n-1} + n\int_{u_0-u}^{\infty} f(\epsilon)^2 F(\epsilon)^{n-2}d\epsilon} \\
&< \frac{1 - F(u_0-u)^n}{nf(u_0-u)F(u_0-u)^{n-1}} \\
&= \Phi^M(u,n).
\end{aligned}$$

Next, I show that $u^O < u^M$. To see this, remember that u^O is given by solving $\partial W(\mathbf{u})/\partial x_i$ for all $i \in \mathcal{N}$, where $\partial W(\mathbf{u})/\partial x_i$ is given by equation (15). This can be rewritten as

$$\Phi^M(u,n)w'(u) + v(u) + \pi(u) - v_0 = 0.$$

Thus, u^O is decreasing in v_0, and u^M equals u^O at $v_0 = 0$. Thus, $u^O < u^M$.

Proof of Proposition 8 I show that when $K \in [\hat{K}, \bar{K})$, welfare is lower with potential entrant than without no potential entrant. The welfare in the case where platform E does not enter and platform I sets consumer utility u_I is given by

$$W_M(u_I) := \left[u_0 + v_0 + \int_{u_0-u_I}^{\infty} (\epsilon + \delta_u + \delta_v - c)f(\epsilon)d\epsilon\right],$$

which has the derivative $W_M'(u_I) = -f(u_0-u_I)(u_0-u_I+\delta_u+\delta_v-c)$. Thus, for all $u_I \leq u_0 + \delta_u + \delta_v - c = u^O$, we have $W_M'(u_I) < 0$. The fact that $\tilde{u}(K) < u^B < u^O$ implies that $W_M(\hat{u}(K)) < W_M(u^B)$, which shows that the welfare is lower with potential entrant than without potential entrant.

Proof of Proposition 9 Applying the implicit function theorem to equation (20), we have

$$\operatorname{sign}\left(u'(\Delta)\right) = \operatorname{sign}\left(-\frac{1}{\tau} - \frac{4}{\tau}\int_{\underline{\epsilon}}^{\infty} f^2(\epsilon)d\epsilon[v(u)+\pi(u)]\right) < 0.$$

Analysis of Example 2 Suppose that $n = 1$ and $F(\epsilon) = 1 - \exp(-\lambda\epsilon)$. Then, the equilibrium condition for x^* can be written as

$$\left[1 - \frac{\overline{ps}(1-\rho_\pi)}{\overline{cs}(1-\rho_u)} - \frac{\kappa}{\theta(u^B)\overline{cs}(1-\rho_u)}\right]\frac{1}{\lambda} + \frac{\overline{ps}[\rho_\pi + \theta(u^B)^2(1-\rho_\pi)]}{2} - (1-\theta(u^B))\kappa = 0.$$

In this case, the likelihood of entry, $1-\theta(u^B)$ is increasing in $\overline{cs}$, decreasing in ρ_u, and increasing in ρ_π. $1-\theta(u^B)$ increases with $\overline{ps}$ if and only if $\kappa > \hat{\kappa}$ for some $\hat{\kappa}$. To see this, let $x := 1-\theta(u^B)$. Then, x increases with κ if and only if

$$-\frac{(1-\rho_\pi)}{\lambda\overline{cs}(1-\rho_u)} + \frac{\rho_\pi + (1-x)^2(1-\rho_\pi)}{2} > 0.$$

Let $\widetilde{x} := 1 - \sqrt{\frac{2}{\lambda\overline{cs}(1-\rho_u)} - \frac{\rho_\pi}{1-\rho_\pi}}$. Then, the above condition can be written as $x < \widetilde{x}$. Such condition holds when $\kappa > \hat{\kappa}$ for some $\hat{\kappa}$. At $\hat{\kappa}$, we have

$$\left[1 - \frac{\hat{\kappa}}{(1-\widetilde{x})\overline{cs}(1-\rho_u)}\right]\frac{1}{\lambda} - \widetilde{x}\hat{\kappa} = 0.$$

$\hat{\kappa}$ is increasing in ρ_π

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Comparisons of Output Subsidy and R&D Subsidy in a Differentiated Market

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Abstract

This study compares Cournot and Bertrand competition with R&D investment under government policies between output and R&D subsidies. We show that firms invest more (less) R&D and the government grants more (less) subsidies under Cournot than Bertrand competition with output (R&D) subsidy policies. We also show that Cournot and Bertrand competitions yield the same welfare with output subsidy while Bertrand yields higher welfare than Cournot with R&D subsidy irrespective of products substitutability. Finally, we show that firms' profits and social welfare are always higher under output subsidies in Cournot competition, while those can be higher under R&D subsidies in Bertrand competition if the products substitutability is high and the firm's R&D investment is efficient.

JEL Classifications: L13, H20

Keywords: Cournot competition; Bertrand competition; R&D investment; Output Subsidy; R&D subsidy

1. Introduction

Comparisons between Cournot and Bertrand competition in a differentiated product duopoly market have been popular in the literature of oligopoly theory since Dixit (1979) and Singh and Vives (1984). Recent studies have also examined the relationship between different market structures and R&D (research and development) activities. Qiu (1997) considered a process R&D with cost-reducing activities and showed that Cournot induces more R&D effort than Bertrand but price is lower and output is larger in Bertrand than Cournot. Kabiraj and Roy (2002) considered different marginal costs and showed that Cournot firms invest a larger amount on R&D than Bertrand firms but Cournot price can be less than Bertrand price when the R&D technology is relatively inefficient. Hinloopen and Vandekerckhove (2009) considered the efficiency of R&D that generates input spillovers and showed that Cournot firms invest more in R&D than Bertrand but Cournot can yield lower prices than Bertrand when the R&D process is efficient, spillovers are substantial, and products are less differentiated. Basak and Wang (2019) also studied R&D competition in a mixed duopoly where the public firm competes with a private firm. They showed that the public firm invests more R&D than the private firm and Bertrand is the equilibrium of endogenous choice between Bertrand and Cournot.

However, all these previous works did not consider the effect of R&D policy. Due to the world-wide trends of globalization and innovations, oligopolistic firms have intensified market competition and thus policy makers have enacted various policies to encourage R&D activities.[1] Recently, a number of studies also have assessed the welfare consequences of R&D activities in the light of governmental intervention.[2] The extensive studies on R&D incentives and policy implications for innovation under imperfect competition are contemporary and practical.

This study examines and compares output and R&D subsidy policies between Cournot and Bertrand competitions with R&D activities. We show that (i) firms invest more (less) R&D and the government grants more (less) subsidies under Cournot than Bertrand competition with output (R&D) subsidy policies; (ii) Cournot and Bertrand competitions yield the same social welfare with output subsidy policies while Bertrand competition yields higher social welfare than Cournot competition with R&D subsidy policies; (iii) firms' profits and social welfare are always higher under output subsidies in Cournot competition, while they can be higher under R&D subsidies in Bertrand competition if the products substitutability is high and the R&D investment of firms is efficient.

The remainder of this paper is organized as follows. In Section 2, we formulate a differentiated duopoly model with R&D where the government can grant an output or an R&D subsidy policy. We compare the equilibrium results with output or R&D subsidy under Cournot and Bertrand competitions

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[1] For example, EU institutions have reaffirmed their commitment to R&D policies and consequently, the budgets of the research Framework Programs (FPs) have grown exponentially, from EUR 3.3 billion in the first FP, launched in 1984, to EUR 80 billion of Horizon 2020. Further, the Research, Innovation and Science Policy Experts (RISE) high-level group, created in 2014, has proposed double this budget or, at least, the maintenance of this growth rate, which would lead to a 7-year budget of more than EUR 120 billion in current prices for the next period. For more details, see Miyagiwa and Ohno (2002), Marinucci (2012), and Chen et al. (2021).

[2] For early discussions, see d'Aspremont and Jacquemin (1988), Kamien et al. (1992), Poyago-Theotoky (1995, 1998, 1999), Lee (1998) and Beath et al. (1998), among others. Recent works extend the analysis into different directions. For example, in a Cournot competition, Yang and Nie (2015) and Lee and Muminov (2020) studied R&D subsidies with asymmetric information while Kesavayuth and Zikos (2013) and Lee et al. (2017) compared output and R&D subsidy policies in a mixed market.

in Section 3 and 4, respectively. In Section 5 we compare the social welfare under the two subsidy policies and discuss policy implications. Finally, we conclude in Section 6.

2. The Basic model

Consider a duopoly market where two firms produce differentiated commodities where a quasi-linear utility function of the representative consumer is denoted by $U = a(q_1 + q_2) - \frac{(q_1{}^2+2bq_1q_2+q_2{}^2)}{2} + m$, where a is a positive constant, m is the consumption of the outside goods, q_i denotes the quantity of the good i, which is produced by the firms, respectively, and $b \in (0,1)$ represents the degree of product substitutability. The inverse demand function is $p_i = a - q_i - bq_j$, $(i, j = 1,2; i \neq j)$, where p_i is the price of good *i*. Then, consumer surplus is $CS = U - p_1q_1 - p_2q_2$.

We assume that firms invest R&D in order to reduce the cost of production. In specific, the cost function for firm *i* is denoted as $C_i = (c - x_i)q_i$ where x_i is the R&D investment of firm *i* and $a > c > x_i > 0$. Each firm has to spend $\Gamma(x_i) = \frac{r}{2}x_i^2$ to implement cost-reducing R&D where the R&D investment causes decreasing returns to scale and *r* represents the efficiency of R&D investment.

We assume that government grants output or R&D subsidies, s^P or s^R, respectively, where the superscript P denotes production output subsidy and the superscript R denotes an R&D subsidy. The profit function for firm i is denoted by $\pi_i = p_iq_i - (c - x_i)q_i - \frac{r}{2}x_i^2 + s^Pq_i + s^Rx_i$. The social welfare is defined as the sum of consumer surplus and firms' profits minus total output subsidy, which is given as: $W = CS + \sum_{i=1}^{2} \pi_i - \sum_{i=1}^{2} s^P q_i - \sum_{i=1}^{2} s^R x_i$. We assume that both firms maximize their own profits while the government maximizes the welfare.[3]

As a benchmark, we can obtain the first-best outcome, which yields that highest welfare from the direct allocation of the output productions and R&D investments.

$$x_i^F = \frac{a-c}{r+br-1},\ q_i^F = \frac{(a-c)r}{r+br-1},\ p_i^F = \frac{-a+(1+b)cr}{r+br-1},\ W^F = \frac{(a-c)^2 r}{r+br-1} \tag{1}$$

Since there is no strategic interaction, i.e., $\frac{\partial q_j}{\partial x_i} = 0\ (i, j = 1,2; i \neq j)$, the first-best R&D allocation is

[3] We will focus on the comparison between an output subsidy of $\{s^P > 0$ and $s^R = 0\}$ and an R&D subsidy of $\{s^P = 0$ and or $s^R > 0\}$ under Cournot or Bertrand competition, respectively. Note that the first-best outcome can be obtained by policy mix of $\{s^P \neq 0$ and $s^R \neq 0\}$. Appendix B provides regularity conditions and detailed analysis for comparisons.

determined at the marginal where $\frac{\partial W}{\partial x_i} = q_i - rx_i = 0$. Notice also that $x_i q_i - \frac{r}{2}x_i^2 = \frac{r}{2}x_i^2$, in which net benefit of R&D in the LHS (total R&D outputs for reducing unit cost minus total R&D expenditures) is positive and equals to the total R&D expenditures in the RHS.

In this analysis, we compare the equilibrium outcomes between Cournot or Bertrand competition under output and R&D subsidies, respectively, when both firms invest R&D. The game structure runs as follows. In the first stage, government grants output or R&D subsidies to maximize the social welfare. In the second stage, both firms decide R&D investment independently and simultaneously to maximize their own profits. In the third stage, both firms compete in Cournot or Bertrand competitions. We solve the subgame perfect Nash equilibrium by backward induction.

3. Analysis with output subsidy policies

We first analyze and compare the equilibrium outcomes under output subsidies between Cournot or Bertrand competition when both firms invest R&D.

3.1 Cournot competition

In the third stage, both firms choose quantities. The equilibrium quantities are as follows:

$$q_i = \frac{2(a - c + s^P + x_i) - b(a - c + s^P + x_j)}{4 - b^2} \tag{2}$$

In the second stage, both firms choose R&D investments. The equilibrium results are as follows:[4]

$$x_i = \frac{4(a - c + s^P)}{(2 - b)(2 + b)^2 r - 4} \tag{3}$$

In the first stage, the maximization of social welfare with respect to s^P yields the following optimal output subsidy:

$$s^{CP} = \frac{(a - c)(16r + b^4 r - 4b^2(1 + 2r))}{E} \tag{4}$$

where $E = 8b^2(1 - r) - 16(1 - r) + 16br - 8b^3 r + b^4 r + b^5 r > 0$ and the superscript "CP" denotes the equilibrium outcomes under production output subsidy policies in Cournot competition.

[4] From the reaction function of each firm under Cournot competition, we can see R&D investments are strategic substitutes for both firms. However, output subsidies monotonically increase both quantities and R&D investments of Cournot firms.

Under the regularity conditions, we have the following equilibrium outcomes:[5]

$$x_i^{CP} = \frac{4(4-b^2)(a-c)}{E} \tag{5}$$

$$q_i^{CP} = \frac{(4-b^2)^2(a-c)r}{E} \tag{6}$$

$$p_i^{CP} = \frac{(1+b)(4-b^2)^2cr - 8a(2-b^2)}{E} \tag{7}$$

$$\pi_i^{CP} = \frac{(4-b^2)^2(a-c)^2r((4-b^2)^2r-8)}{E^2} \tag{8}$$

$$W^{CP} = \frac{(4-b^2)^2(a-c)^2r}{E} \tag{9}$$

We compare the results with no subsidy where the superscript "C" denotes the equilibrium outcomes under no subsidy in Cournot competition, and the first-best outcome, where the superscript "F" denotes the optimal levels that the government directly determines to maximize the welfare.[6] Figure 1 shows the graphical relations between equilibria under Cournot competition.[7]

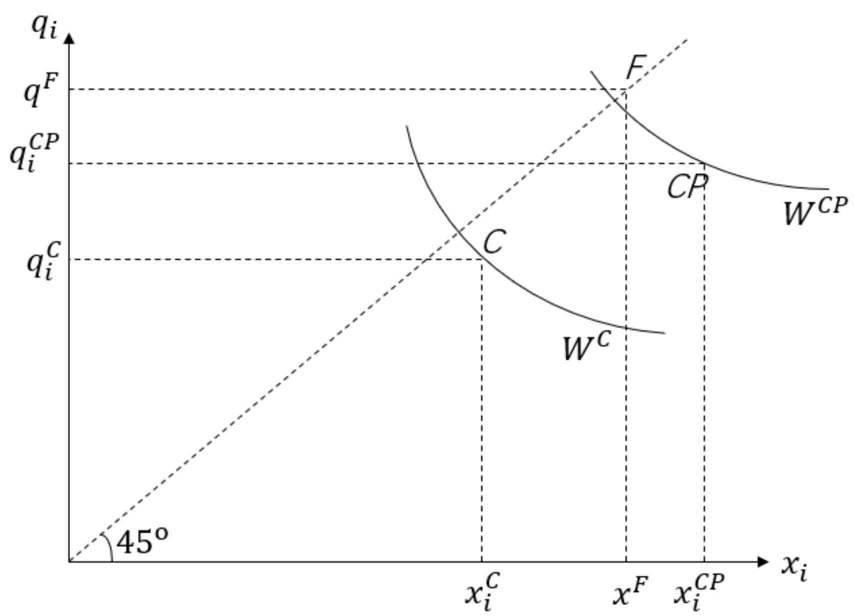

Figure 1: output subsidy vs. no subsidy under Cournot competition

[5] In Appendix A, according to Hinloopen and Vandekerckhove (2009), we provide regularity conditions under Cournot or Bertrand competition where the second-order conditions, positive post-innovation costs and the stable equilibrium are examined.

[6] Some necessary proofs of propositions and lemmas are provided in Appendix C while others are straight-forward and thus omitted.

[7] In the followings, we set $r = 1$ and $a - c = 1$ in all the figures.

Lemma 1. $x_i^{CP} > x_i^F > x_i^C$, $q_i^F > q_i^{CP} > q_i^C$, and $W^F > W^{CP} > W^C$ for $b \in (0,1)$.

It states that output subsidies increase not only R&D investments but outputs of the firms, which increase social welfare, compared to no subsidy, but induce under-production and over-investment to Cournot firms, compared to the first-best, which results in welfare loss. This is because the strategic effect of R&D to the output is positive under Cournot competition, i.e., $\frac{\partial q_j}{\partial x_i}\frac{\partial \pi_i}{\partial q_j} > 0$ $(i,j = 1,2; i \neq j)$, while the outputs are strategic substitutes.[8]

3.2 Bertrand competition

In the third stage, both firms choose price where the demand function for firm i is given as $q_i = \frac{a-ab-p_i+bp_j}{1-b^2}$. The equilibrium prices are as follows:

$$p_i = \frac{a(2-b-b^2)+(2+b)c-2s^P-bs^P-2x_i-bx_j}{4-b^2} \tag{10}$$

In the second stage, both firms choose R&D investments. The equilibrium results are as follows:[9]

$$x_i = \frac{2(2-b^2)(a-c+s^P)}{b^2(2-6r)+8r+4br-b^3r+b^4r-4} \tag{11}$$

In the first stage, the maximization of social welfare with respect to s^P yields the following optimal output subsidy:

$$s^{BP} = \frac{(a-c)(16r-b^4(2-9r)-b^6r+4b^2(1-6r))}{E} \tag{12}$$

where the superscript "BP" denotes the equilibrium outcomes under output subsidy policies in Bertrand competition.

Under the regularity conditions, we have the following equilibrium outcomes:

$$x_i^{BP} = \frac{2(4-b^2)(2-b^2)(a-c)}{E} \tag{13}$$

$$q_i^{BP} = \frac{(4-b^2)^2(a-c)r}{E} \tag{14}$$

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8 See Qiu (1997) and Hinloopen and Vandekerckhove (2009) for discussion on the strategic effects of R&D between Cournot and Bertrand competitions. The positive strategic effect leads to over-investment in the absence of spillovers, see Brander and Spencer (1983).

9 From the reaction function of each firm under Bertrand competition, we can see R&D investments are strategic substitutes for both firms. However, output subsidies monotonically increase both quantities and R&D investments of Bertrand firms.

$$p_i^{BP} = \frac{(1+b)(4-b^2)^2cr - 8a(2-b^2)}{E} \tag{15}$$

$$\pi_i^{BP} = \frac{(4-b^2)^2(a-c)^2r(b^4(2-9r) - 8 + 16r - b^6r + 8b^2(1-3r))}{E^2} \tag{16}$$

$$W^{BP} = \frac{(4-b^2)^2(a-c)^2r}{E} \tag{17}$$

We compare the results with no subsidy where the superscript "B" denotes the equilibrium outcomes under no subsidy in Cournot competition, and the first-best outcome. Figure 2 shows the graphical relations between equilibria under Bertrand competition.

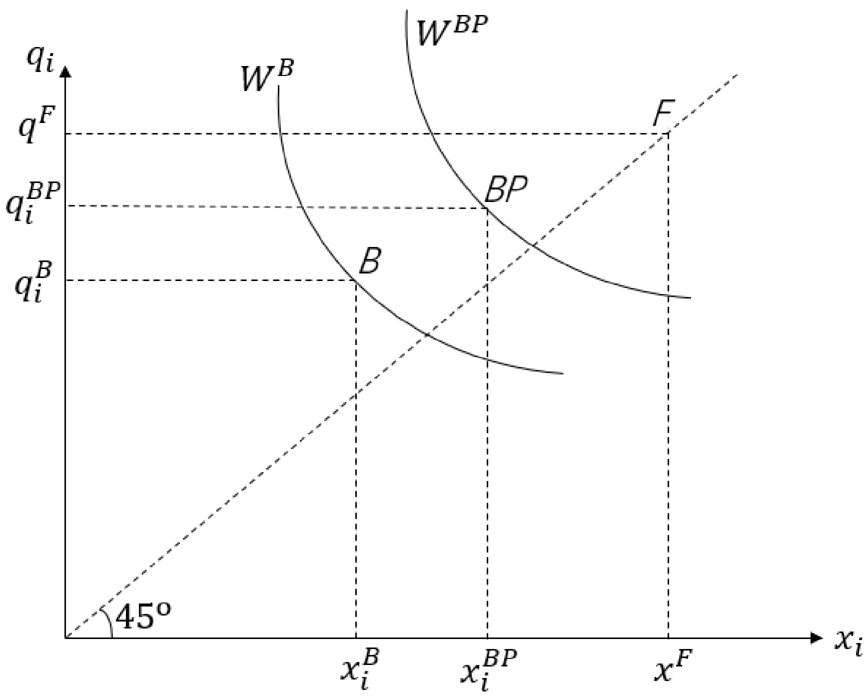

Figure 2: output subsidy vs. no subsidy under Bertrand competition

Lemma 2. $x_i^F > x_i^{BP} > x_i^B$, $q_i^F > q_i^{BP} > q_i^B$, and $W^F > W^{BP} > W^B$ for $b \in (0,1)$.

It states that output subsidies increase not only R&D investments but outputs of the firms, which increase social welfare, compared to no subsidy, but induce both under-production and under-investment to Bertrand firms, compared to the first-best, which results in welfare loss. This is because the strategic effect of R&D to the output is negative under Bertrand competition, i.e., $\frac{\partial p_j}{\partial x_i}\frac{\partial \pi_i}{\partial p_j} < 0$ $(i,j = 1,2; i \neq j)$, while the prices are strategic complements.

3.3 Comparisons

We compare the Cournot and Bertrand competitions under output subsidies. Figure 3 combines Figure 1 and 2, and shows the graphical relations between Cournot and Bertrand competitions. It represents that output subsidies increase R&D investments and outputs in both Cournot and Bertrand firms, compared to no subsidies, but the R&D investments exceeds the first-best in Cournot but less than the first-best in Bertrand competition.

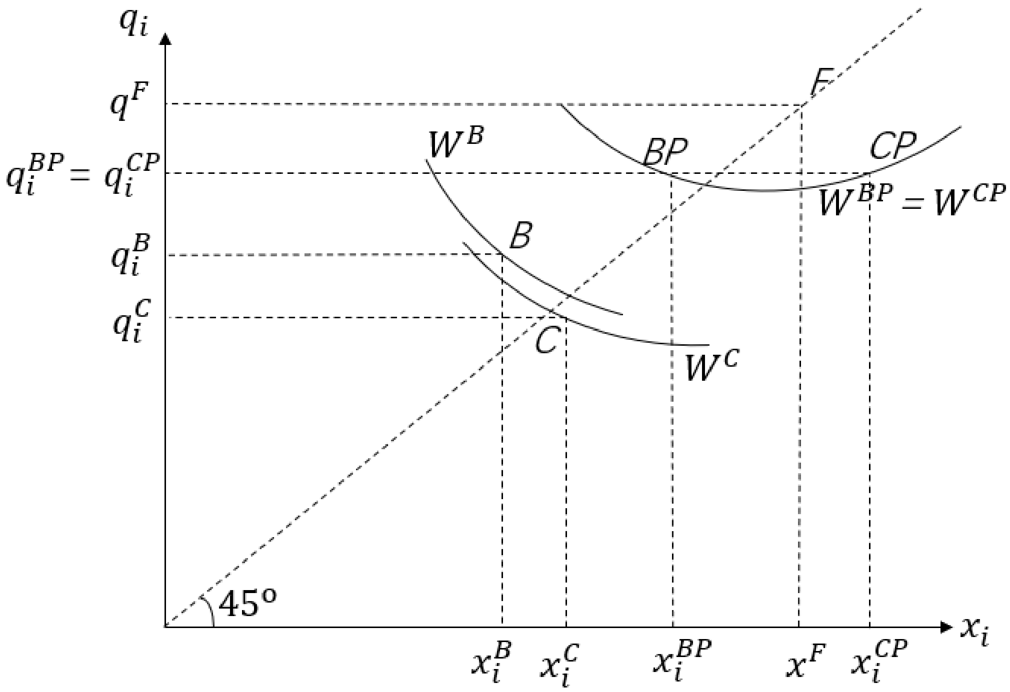

Figure 3: Welfare comparisons under output subsidies

Proposition 1. $s^{CP} > s^{BP} > 0$ for $b \in (0,1)$.

It states that the government grants more output subsidies to Cournot firms than Bertrand firms. In the absence of output subsidies, it is well-known that Cournot firms produce less outputs than Bertrand firms due to the strategic effects between quantities and prices, but undertake more R&D.[10] Under quantity competition with strategic substitutes, output subsidies can induce Cournot firms to undertake R&D more aggressively, which reduces its marginal cost and thus a firm can increase outputs which in turn decreases the output of rival firm and increase its profit. Under price competition with strategic complements, if a Bertrand firm with output subsidies aggressively undertakes R&D, which reduces its

[10] For more explanations on the strategic effects, see Singh and Vives (1984) for the case without R&D and Qiu (1997) for the case with R&D activities.

marginal cost, then it increases its outputs and reduces its price which in turn reduces the price of rival firm. Then, more R&D can reduce the profit of the firm by undercutting the prices. Therefore, the government has an incentive to provide more output subsidy to Cournot firms, which can affect allocation efficiency by encouraging output production decisions.

Proposition 2. $x_i^{CP} > x_i^{BP}$ for $b \in (0,1)$.

It states that Cournot firms undertake more R&D and thus lower production cost under output subsidies. This finding confirms the previous result in the absence of output subsidies (Qiu, 1997; Hinloopen and Vandekerckhove, 2009). In the presence of output subsidy, it is interesting to see that both Cournot and Bertrand firms' incentives to undertake more R&D investments but that of Cournot firms is much higher, which can also induce over-investment, compared to the first-best, to reduce its cost and produce more outputs to earn more profits.

Proposition 3. $q_i^{CP} = q_i^{BP}$ and $p_i^{CP} = p_i^{BP}$ for $b \in (0,1)$.

It is interesting to find that output subsidies make outputs and prices be the same under Cournot and Bertrand firms even though the government induces Cournot firms to undertake more R&D. We can explain these results. First, it is noteworthy that when firms do not invest R&D, the quantities and prices of firms are the same under both Cournot and Bertrand competitions if there are optimal output subsidy policies and both firms have the same cost.[11] Second, we can show that the net benefits from R&D between Cournot and Bertrand firms are the same and positive under the output subsidies, i.e., $x_i^{CP} q_i^{CP} - \frac{r}{2}\left(x_i^{CP}\right)^2 = x_i^{BP} q_i^{BP} - \frac{r}{2}\left(x_i^{BP}\right)^2$. This is because output subsidies have opposite effects on the R&D decisions of the firms, i.e., the strategic effect of R&D to the Cournot firms is positive, i.e., $\frac{\partial q_j}{\partial x_i}\frac{\partial \pi_i}{\partial q_j} > 0$ while the effect of R&D to the Bertrand firms is negative, i.e., $\frac{\partial p_j}{\partial x_i}\frac{\partial \pi_i}{\partial p_j} < 0$. Since both effects are off-set so that the net benefits from R&D are the same under the output subsidies. This results in that the quantities and prices can be also the same in both Cournot and Bertrand competitions.[12]

[11] Regarding the efficiency properties of output subsidies, Kim and Lee (1995) and Lee (1997) analyzed the different oligopolistic incentives under asymmetric information and showed that output subsidies can still obtain the first-best allocation.

[12] Note that this result holds under the linear marginal cost between the firms. If we consider a quadratic cost

Therefore, output subsidies can rearrange firms' production efficiency to redistribute the allocations of firms' outputs, given different levels of R&D investments.[13] This result also implies that consumer surplus under Cournot firms are same as that under Bertrand firms.

Proposition 4. $\pi_i^{CP} > \pi_i^{BP}$ for $b \in (0,1)$.

It states that Cournot firm gains more profit since it undertakes more R&D (to reduce the unit cost) and earn more output subsidy, which induces the less unit cost of output production under the same prices and quantities with Bertrand firms. This finding implies that irrespective of product substitutability, Cournot competition can be an equilibrium if firms can choose market mode between quantity and price competitions.[14]

Proposition 5. $W^{CP} = W^{BP}$ for $b \in (0,1)$.

It is also interesting to find that both Cournot and Bertrand firms yield the same welfare under output subsidies. From Proposition 3, the same consumer surplus between both Cournot and Bertrand competitions while the effect of output subsidies to the profits of both firms is exactly off-set in the social welfare even though Cournot firm gains more profit.

4. Analysis with R&D subsidy policies

We also analyze and compare the equilibrium outcomes under R&D subsidies between Cournot or Bertrand competition when both firms invest R&D.

4.1 Cournot competition

In the third stage, the equilibrium quantities are as follows:

$$q_i = \frac{(a-c)(2-b) + 2x_i - bx_j}{4-b^2} \tag{18}$$

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between the firms, however, it does not hold even under output subsidies.

13 Note also that under the same R&D activities, e.g., if $x_i = x_i^F$, output subsidies can attain the first-best outputs irrespective of Cournot or Bertrand competition. However, under the different R&D activities, output subsidies can yield the same output which is lower than the first-best outputs irrespective of Cournot or Bertrand competition.

14 See Häckner (2000), Symeonidis (2003), Matsumura and Ogawa (2012), and Basak (2017) for some related discussions on the endogenous choice of market structure.

In the second stage, the equilibrium R&D investments are as follows:[15]

$$x_i = \frac{4(a-c) + (2-b)(2+b)^2 s^R}{(2-b)(2+b)^2 r - 4} \tag{19}$$

In the first stage, the maximization of social welfare with respect to s^R yields the output subsidy as follows:

$$s^{CR} = \frac{(1-b)(2+b)(a-c)r}{(2-b)M} \tag{20}$$

where the superscript "CR" denotes the equilibrium outcomes under R&D subsidy policies in Cournot competition and $M = 4r + b^2 r - b(1-4r) - 3 > 0$.

Then the resulting equilibrium outcomes are given as follows:

$$x_i^{CR} = \frac{(3+b)(a-c)}{M} \tag{21}$$

$$q_i^{CR} = \frac{(2+b)(a-c)r}{M} \tag{22}$$

$$p_i^{CR} = \frac{(2+3b+b^2)cr + a(-3+b(-1+r)+2r)}{M} \tag{23}$$

$$\pi_i^{CR} = \frac{(a-c)^2 r(6+b(5-8r)-16r+4b^2(1+r)+b^3(1+2r))}{2(-2+b)M^2} \tag{24}$$

$$W^{CR} = \frac{(3+b)(a-c)^2 r}{M} \tag{25}$$

Figure 4 shows the graphical relations between equilibria under Cournot competition.

[15] From the reaction function of each firm under Cournot competition, we can see R&D investments are strategic substitutes for both firms. However, R&D subsidies monotonically increase both quantities and R&D investments of Cournot firms.

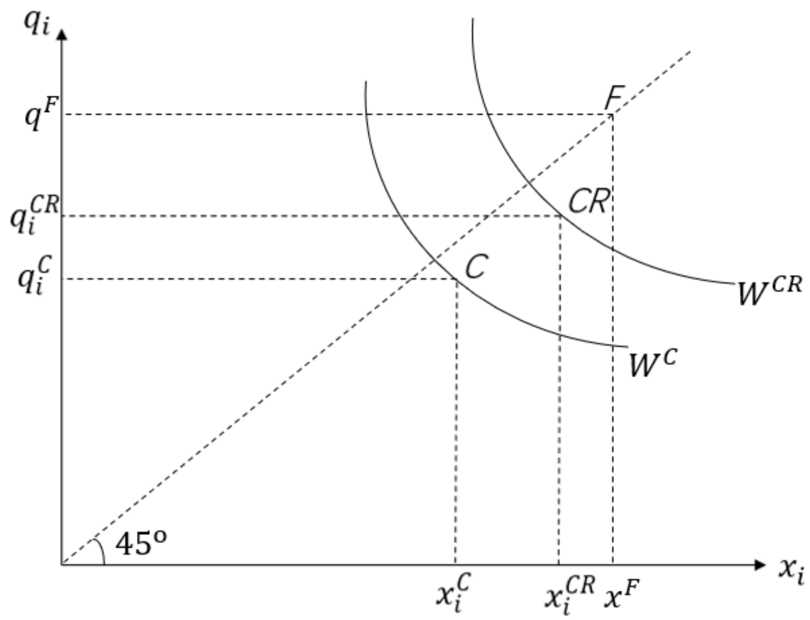

Figure 4: R&D subsidy vs. no subsidy under Cournot competition

Lemma 3. $x_i^F > x_i^{CR} > x_i^C$, $q_i^F > q_i^{CR} > q_i^C$, and $W^F > W^{CR} > W^C$ for $b \in (0,1)$.

It states that R&D subsidies increase not only R&D investments but outputs of the firms, which increase social welfare, compared to no subsidy, but induce both under-production and under-investment to Cournot firms, compared to the first-best, which results in welfare loss. This is also because the strategic effect of R&D to the output is positive under Cournot competition, i.e., $\frac{\partial q_j}{\partial x_i}\frac{\partial \pi_i}{\partial q_j} > 0$ $(i, j = 1,2; i \neq j)$, while the outputs are strategic substitutes.

4.2 Bertrand competition

In this case, the equilibrium prices are as follows:

$$p_i = \frac{a(2-b-b^2) + (2+b)c - 2x_1 - bx_2}{4-b^2} \tag{26}$$

In the second stage, the equilibrium R&D investments are as follows:[16]

$$x_i = \frac{2(2-b^2)(a-c) + (2-b)^2(2+3b+b^2)s^R}{b^2(2-6r) + 8r + 4br - b^3r + b^4r - 4} \tag{27}$$

[16] From the reaction function of each firm under Bertrand competition, we can see R&D investments are strategic substitutes for both firms. However, output subsidies monotonically increase both quantities and R&D investments of Bertrand firms.

In the first stage, the maximization of social welfare with respect to s^R yields the output subsidy:

$$s^{BR} = \frac{(2-b)(a-c)r}{(2+b)H} \tag{28}$$

where the superscript “BR” denotes the equilibrium outcomes under R&D subsidy policies in Bertrand competition and $H = 2b + 4r - 3b^2r + b^3r - 3 > 0$.

The resulting equilibrium outcomes are given as follows:

$$x_i^{BR} = \frac{(3-2b)(a-c)}{H} \tag{29}$$

$$q_i^{BR} = \frac{(2-b)(a-c)r}{H} \tag{30}$$

$$p_i^{BR} = \frac{(2+b-b^2)cr + a(-3+b(2-r)+2r-2b^2r+b^3r)}{H} \tag{31}$$

$$\pi_i^{BR} = \frac{(a-c)^2r(16r+4b^4r-2b^5r+4b^2(2-5r)+b(1-8r)-6-b^3(4-10r))}{2(2+b)H^2} \tag{32}$$

$$W^{BR} = \frac{(3-2b)(a-c)^2r}{H} \tag{33}$$

Figure 5 shows the graphical relations between equilibria under Bertrand competition.

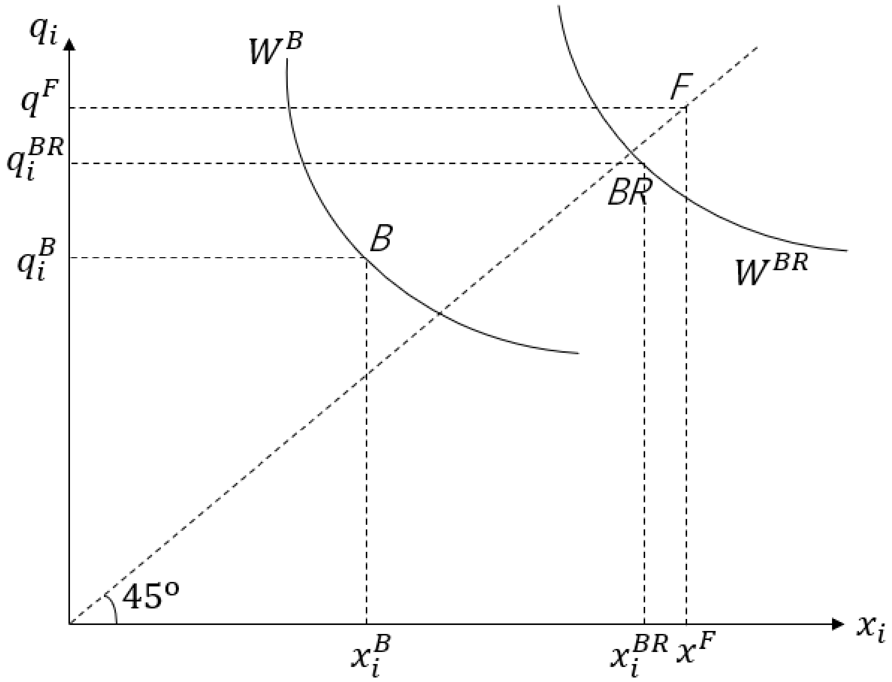

Figure 5: R&D subsidy vs. no subsidy under Bertrand competition

Lemma 4. $x_i^F > x_i^{BR} > x_i^B$, $q_i^F > q_i^{BR} > q_i^B$ and $W^F > W^{BR} > W^B$ for $b \in (0,1)$.

It states that R&D subsidies increase not only R&D investments but outputs of the firms, which increase social welfare, compared to no subsidy, but induce both under-production and under-investment to Bertrand firms, compared to the first-best, which results in welfare loss. This is also because the strategic effect of R&D to the output is negative under Bertrand competition, i.e., $\frac{\partial p_j}{\partial x_i}\frac{\partial \pi_i}{\partial p_j} < 0$ $(i,j = 1,2; i \neq j)$, while the prices are strategic complements.

4.3 Comparisons

We compare the Cournot and Bertrand competitions under R&D subsidies. Figure 6 combines Figure 4 and 5, and shows the graphical relations between Cournot and Bertrand competitions. It represents that R&D subsidies increase R&D investments and outputs in both Cournot and Bertrand firms, compared to no subsidies. With R&D subsidy policies, however, the R&D investments are less than the first-best in both Cournot and Bertrand competition.

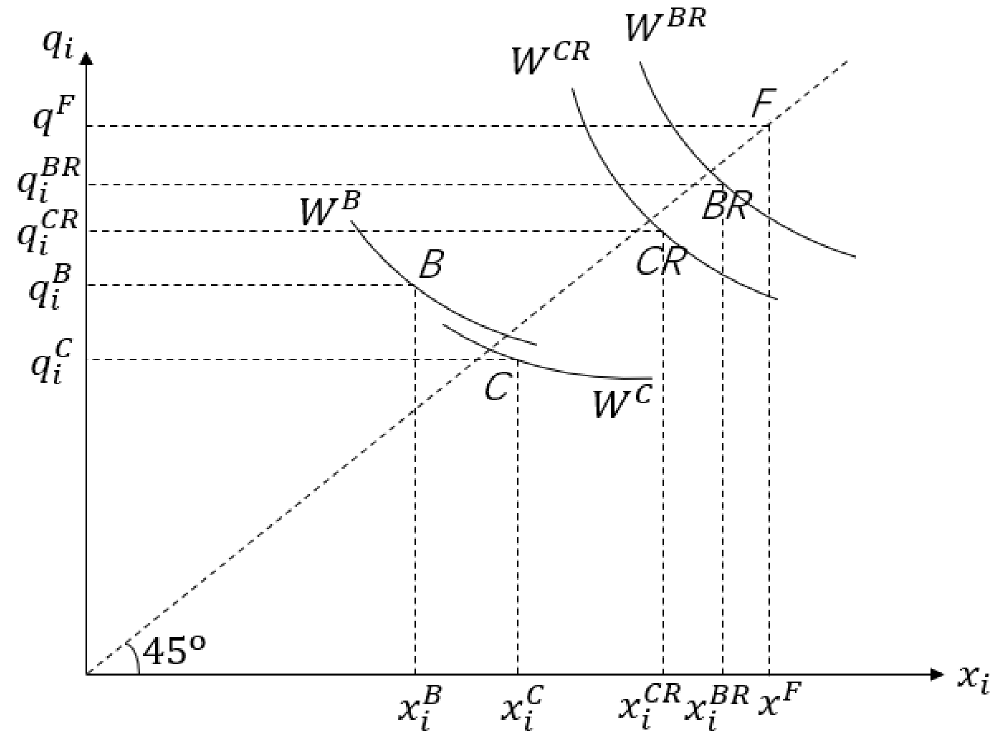

Figure 6: Welfare comparisons under R&D subsidies

Proposition 6. $s^{BR} > s^{CR} > 0$ for $b \in (0,1)$.

It states that the government grants less R&D subsidies to Cournot firms than Bertrand firms. This is contrary to the results with output subsidy policies (Proposition 1). In the absence of R&D subsidies, Bertrand firms invest lower R&D than Cournot firms, which are also lower than the first-best. In order to increase R&D directly, contrary to output subsidy policies, the government has an incentive to provide more R&D subsidies to Bertrand firms to increase social welfare.

Proposition 7. $x_i^{CR} < x_i^{BR}$ for $b \in (0,1)$.

It states that Bertrand firms undertake more R&D and thus lower production cost under R&D subsidies. This is also contrary to the results with output subsidy policies (Proposition 2). Even though Bertrand firm can reduce its cost and produce more outputs to earn more profits under R&D subsidies, it chooses under-investment, compared to the first-best.

Proposition 8. $q_i^{CR} < q_i^{BR}$ and $p_i^{CR} > p_i^{BR}$ for $b \in (0,1)$.

It states that R&D subsidies can induce Bertrand firms to produce more outputs and less prices than those of Cournot firms since the government induces Bertrand firms to undertake more R&D. This is also contrary to the results with output subsidy policies (Proposition 3). Note that $0 < x_i^{CR} q_i^{CR} - \frac{r}{2}\left(x_i^{CR}\right)^2 < x_i^{BR} q_i^{BR} - \frac{r}{2}\left(x_i^{BR}\right)^2$. Even though R&D output subsidies have opposite effects on the R&D decisions of the firms, both effects are not off-set so that the net benefits from R&D with Bertrand firms higher than that with Cournot firms. This results in that outputs (prices) with Bertrand firm higher (lower) than those with Cournot firms under R&D subsidies. This result also implies that consumer surplus is higher under Bertrand competition.

Proposition 9. If $b > b_2$, then $\pi_i^{CR} > \pi_i^{BR}$; however, if $b < b_2$, then there exists $\bar{r}$ so that $\pi_i^{BR} \underset{<}{\overset{>}{=}} \pi_i^{CR}$ if $r \underset{>}{\overset{<}{=}} \bar{r}$, where b_2 is provided in the Appendix.

It states that the profits of Cournot firms are higher than Bertrand firms if the product substitutability is high, while it will be lower than Bertrand firms if the product substitutability is low and R&D investment of firms is relatively efficient. It implies that depending on the substitutability and the efficiency of R&D investment, Cournot or Bertrand competition can be an equilibrium if both firms choose competition mode between quantity and price competitions.

We can explain these results. First, note that both output and R&D subsidies increase the outputs and

R&D investments of the firms under both Cournot and Bertrand competition. On the one hand, we have that $\frac{\partial s^{CR}}{\partial b} < 0$ and $\frac{\partial q_i^{CR}}{\partial b} < 0$ for any b. Thus, lower substitutability increases output subsidy, which increases the outputs of Cournot firms. However, Bertrand firms are more sensitive to the product substitutability, which induces Bertrand firms to set a lower price than Cournot firms. Therefore, if the product substitutability is high enough, Cournot firms will earn more profit than Bertrand firms.[17] On the other hand, it can be showed that $\frac{\partial s^{BR}}{\partial b} \gtrless 0$ and $\frac{\partial q_i^{BR}}{\partial b} \gtrless 0$ if $b \gtrless b_2$. Thus, if the product substitutability is high (for a larger *b*), higher substitutability increases R&D subsidy, which increases the output of Bertrand firms. Therefore, if the product substitutability is high enough, Bertrand firms to set a lower price than Cournot firms and Cournot firms will earn more profit than Bertrand firms. However, if the product substitutability is low (for a smaller *b*), then higher substitutability decreases R&D subsidies, which also decrease the outputs of both Cournot and Bertrand firms. Still Bertrand firms have a higher subsidy (see proposition 6), if firms invest R&D efficiently (lower *r*), then Bertrand firms will earn more profit than Cournot firms. In contrary, if firms invest R&D inefficiently (higher *r*), then Bertrand firms will face more cost loss because they invest more R&D than Cournot firms (see proposition 7). Therefore, Cournot firms will earn more profit again than Bertrand firms if R&D investment is inefficient.

Proposition 10. $W^{CR} < W^{BR}$ for $b \in (0,1)$.

It states that Bertrand competition yields higher social welfare than Cournot with R&D subsidy policies. This is also contrary to the results with output subsidies (Proposition 5). This is because consumer surplus is always higher under Bertrand competition, which overweigh the profit changes. This finding implies that irrespective of product substitutability, the society is better off under Bertrand competition if the government can choose market mode between quantity and price competitions.

5. Discussions

We now compare the equilibrium outcomes between output and R&D subsidy policies with Cournot or Bertrand firms, respectively.

[17] See also Tremblay and Tremblay (2011), Correa-Lopez and Naylor (2004), among others.

Proposition 11.

(i) $s^{CP} > s^{CR}$ and $s^{BP} \gtreqless s^{BR}$ if $b \lesseqgtr b_3 \equiv \frac{4a+c-\sqrt{16a^2-8ac+9c^2}}{2c}$.

(ii) $x_i^{CP} > x_i^{CR}$ and $x_i^{BP} \gtreqless x_i^{BR}$ if $b \lesseqgtr b_3$;

(iii) $q_i^{CP} > q_i^{CR}$ and $q_i^{BP} > q_i^{BR}$ for $b \in (0,1)$.

It states that under Cournot competition the government always sets higher output subsidy to Cournot firms, compared to R&D subsidies, which induce higher outputs and R&D investments to the Cournot firms. However, under Bertrand competition, the government sets higher output subsidy to Bertrand firms if the product substitutability is low. Thus, Bertrand firms always produce more output under output subsidies but they might undertake less R&D under output subsidy if the product substitutability is high.

Finally, we compare the profits and welfares between output and R&D subsidies under Cournot or Bertrand competition, respectively. It is difficult to find the explicit ranges that support regularity conditions and thus we provide a simulation in Appendix and test the specific numbers as examples. From the numerical simulation, we can provide the following propositions.

Proposition 12.

(i) $\pi_i^{CP} > \pi_i^{CR}$ for $b \in (0,1)$ and $\pi_i^{BP} > \pi_i^{BR}$ if $b < b_3$. However, if $b > b_3$, then there exists $\hat{b} > b_3$ and $\hat{r}$ such that $\pi_i^{BP} < \pi_i^{BR}$ if $b > \hat{b}$ and $r < \hat{r}$, while $\pi_i^{BP} > \pi_i^{BR}$ if $b < \hat{b}$ and $r > \hat{r}$.

(ii) $W^{CP} > W^{CR}$ for $b \in (0,1)$ and $W^{BP} > W^{BR}$ if $b < b_3$. However, if $b > b_3$, then there exists a large $\hat{b} > b_3$ and $\hat{r}$ so that $W^{BP} < W^{BR}$ if $b > \hat{b}$ and $r < \hat{r}$, while $W^{BP} > W^{BR}$ if $b < \hat{b}$ and $r > \hat{r}$.

Propositions 12 state that output subsidy policies to Cournot firms always yield higher profits and social welfare than R&D subsidy policies. This result with quantity competition confirms the finding in Kesavayuth and Zikos (2013) and Lee et al. (2017) who examined Cournot duopoly with homogeneous products but the firms' objectives might be different. However, Bertrand firms in price competition shows that the effect of the subsidy policies depends on the products substitutability and the efficiency of R&D investment. When the substitutability is high and the R&D investment of firms is efficient (lower r), less marginal cost is more beneficial and thus, R&D subsidy policies play a more important role to improve firms' profits and the social welfare. However, when the products substitutability is low and the R&D investment of firms is inefficient (higher r), more production is more beneficial and thus, output subsidy policies can improve firms' profits and the social welfare.

6. Conclusions

This study examined and compared the effects of output and R&D subsidy policies on the competition mode between Cournot and Bertrand in a differentiated product duopoly market. We showed that firms invest more R&D and the government grants more subsidies under Cournot than Bertrand with output subsidies, but the results are reversed with R&D subsidies. We also showed that firms earn more profits under Cournot than Bertrand with output subsidy while the profits of firms are higher (lower) under Cournot than Bertrand if the efficiency of firms' R&D investment is high (low) with R&D subsidy. As a result, the social welfares are the same in both Cournot than Bertrand competitions with output subsidy policies, while the social welfare is always lower under Cournot than Bertrand competition. Finally, we show that firms' profits and social welfare are always higher under output subsidies in Cournot competition, while they can be higher under R&D subsidies in Bertrand competition if the products substitutability is high and the R&D investment of firms is efficient.

There are some topics for future research. First, we can extend this model into an oligopolistic competition.[18] Second, we can consider the effect of R&D spillovers. Finally, we can examine a mixed market where the objectives between the firms are different.[19] We leave them for further research.

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[18] For example, see Hsu and Wang (2005), Tremblay and Tremblay (2011), Haraguchi and Matsumura (2016), among others.

[19] Recent works have also analyzed different objectives of the firms with/without subsidy policies. For example, see Gil-Molto et al. (2011), Matsumura et al. (2013), Lee and Tomaru (2017), and Leal et al. (2021).

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Appendix A. Regularity conditions

According to Hinloopen and Vandekerckhove (2009), we consider regularity conditions which include the second-order conditions, positive post-innovation costs and the stable equilibrium.

First, under output subsidy policies, the second-order conditions under Cournot and Bertrand competition require, respectively:

$$r > \frac{8}{(4-b^2)^2} \tag{R1}$$

$$r > \frac{2(2-b^2)^2}{(4-b^2)^2(1-b^2)} \tag{R2}$$

Positive post-innovation costs ($c - x_i > 0$) under Cournot and Bertrand competition require, respectively:

$$r > \frac{4(a(4-b^2)-b^2c)}{(1+b)(4-b^2)^2c} \tag{R3}$$

$$r > \frac{2(2-b^2)(a(4-b^2)+b^2c)}{(1+b)(4-b^2)^2c} \tag{R4}$$

Stability conditions under Cournot competition require:

$$r > \frac{4}{(2-b)^2(2+b)} \tag{R5}$$

$$r > \frac{8a(2-b^2)}{(1+b)(4-b^2)^2c} \tag{R6}$$

Stability conditions under Bertrand competition require:

$$r > \frac{2(2-b^2)}{(2+b)^2(2-3b+b^2)} \tag{R7}$$

$$r > \frac{8a(2-b^2)}{(1+b)(4-b^2)^2c} \tag{R8}$$

Second, under R&D subsidy policies, the second-order conditions under Cournot and Bertrand competition require, respectively:

$$r > \frac{8}{(4-b^2)^2} \tag{R9}$$

$$r > \frac{2(2-b^2)^2}{(4-b^2)^2(1-b^2)} \tag{R10}$$

Positive post-innovation costs $(c - x_i > 0)$ under Cournot and Bertrand competition require, respectively:

$$r > \frac{a(3+b)}{(2+b)^2c} \tag{R11}$$

$$r > \frac{a(3-2b)}{(-2+b)^2(1+b)c} \tag{R12}$$

Stability conditions under Cournot competition require:

$$r > \frac{4}{(2-b)^2(2+b)} \tag{R13}$$

$$r > \frac{a(3+b)}{(2+b)(a+c+bc)} \tag{R14}$$

Stability conditions under Bertrand competition require:

$$r > \frac{2(2-b^2)}{(2+b)^2(2-3b+b^2)} \tag{R15}$$

$$r > \frac{a(3-2b)}{(2+b-b^2)(a(1-b)+c)} \tag{R16}$$

Appendix B. The First-best outcome and comparisons

First, the first-best outcome can be directly obtained by the optimal choices of outputs and R&D investments from the welfare function:

$$x_i^F = \frac{a-c}{r+br-1},\ q_i^F = \frac{(a-c)r}{r+br-1},\ p_i^F = \frac{-a+(1+b)cr}{r+br-1},\ W^F = \frac{(a-c)^2r}{r+br-1}$$

Then, regularity conditions under the first-best are as follows:

(B1) The second-order condition require: $r > \frac{1}{1-b^2}$ (R17)

(B2) Positive post-innovation costs require: $r > \frac{a}{c+bc}$ (R18)

(B3) Stability conditions require: $r > \frac{1}{1-b}$ (R19)

Second, we compare the equilibrium outcomes with the first-best outcome in Figure 3 and 6, respectively. According to R17 to R19, we obtain the ranges of r and b when comparing equilibrium outcomes with first-best outcomes: $r > \frac{a}{c+bc}$ if $b \leq \frac{a-c}{a+c}$; $r > \frac{1}{1-b}$ if $b > \frac{a-c}{a+c}$. With output subsidies in Figure 3, it is straight-forward to show: $q_i^F > q_i^{CP} = q_i^{BP} > q_i^B > q_i^C$ while $x_i^{CP} > x_i^F > x_i^{BP} > x_i^C > x_i^B$. With R&D subsidies in Figure 6, it is also straight-forward to show: $q_i^F > q_i^{BR} > q_i^{CR} > q_i^B > q_i^C$ while $x_i^F > x_i^{BR} > x_i^{CR} > x_i^C > x_i^B$.

Finally, we confirm that policy mix of output and R&D subsidies can achieve the first-best outcomes in Cournot and Bertrand competitions, respectively. On the one hand, the policy mix equilibrium outcomes in Cournot competition are as follows:

$$s_M^{CP} = \frac{(a-c)r}{-1+r+br} > 0,\ s_M^{CR} = \frac{-b^2(a-c)r}{(4-b^2)(r+br-1)} < 0$$

$$x_{iM}^C = \frac{a-c}{-1+r+br} = x_i^F,\ q_{iM}^C = \frac{(a-c)r}{-1+r+br} = q_i^F,\ p_{iM}^C = \frac{-a+(1+b)cr}{-1+r+br} = p_i^F,\ W_M^C = \frac{(a-c)^2 r}{-1+r+br} = W^F$$

where subscript "M" denotes the outcomes with policy mix. Note that the government grants output subsidies but levies R&D taxes in Cournot competition.[20]

On the other hand, the policy mix equilibrium outcomes in Bertrand competition are as follows:

$$s_M^{BP} = \frac{(a-c)r}{-1+r+br} > 0,\ s_M^{BR} = \frac{b^2(a-c)r}{(4-b^2)(r+br-1)} > 0$$

$$x_{iM}^B = \frac{a-c}{-1+r+br} = x_i^F,\ q_{iM}^B = \frac{(a-c)r}{-1+r+br} = q_i^F,\ p_{iM}^B = \frac{-a+(1+b)cr}{-1+r+br} = p_i^F,\ W_M^B = \frac{(a-c)^2 r}{-1+r+br} = W^F$$

Note that the government grants both output and R&D subsidies in Bertrand competition.

Appendix C. The Proofs

First, with output subsidies, by comparing R1 to R8, we obtain the ranges of *r* and *b* when comparing equilibrium outcomes: $r > \frac{4(a(4-b^2)-b^2c)}{(1+b)(4-b^2)^2c}$ if $b \leq b_1$; $r > \frac{2(2-b^2)}{(2+b)^2(2-3b+b^2)}$ if $b > b_1$, where

$$b_1 = \frac{6a+9c+\sqrt{6}c\sqrt{16-\frac{16a}{c}+\frac{3(2a+3c)^2}{c^2}-\frac{2(2^{2/3}(26a^2-10ac+35c^2))}{cM^{1/3}}-\frac{2(2^{1/3}M^{1/3})}{c}-\frac{3\sqrt{3}(2a+7c)(4a^2-4ac+5c^2)}{c^3Y}}-\sqrt{3}cY}{12c}$$

[20] These results are also provided in Zikos (2007) and Lee and Tomaru (2017) who examined Cournot competition with a policy mix in a mixed oligopoly market where the firms have different objectives in producing homogeneous products.

$Y = \sqrt{\frac{8(a-c)c-24c(a-c)+3(2a+3c)^2+\frac{4(2^{2/3}c(26a^2-10ac+35c^2))}{M^{1/3}}+4(2^{1/3}cM^{1/3})}{c^2}}$ and $M = (222a^2c - 140a^3 - 465ac^2 + 383c^3 - 3\sqrt{3}\sqrt{812a^4c^2 - 576a^6 - 800a^5c - 7500a^3c^3 + 6451a^2c^4 - 10470ac^5 + 2257c^6})$

Then we have the following proofs:

Proof of lemma 1.

If we set $s^P = 0$ in (2) and (3), then we have the following equilibrium outcomes:

$$x_i^C = \frac{4(a-c)}{(2-b)(2+b)^2r-4} \tag{34}$$

$$q_i^C = \frac{(2-b)(2+b)(a-c)r}{(2-b)(2+b)^2r-4} \tag{35}$$

$$p_i^C = \frac{(b^2+b^3-4-4b)cr+a(4-(4-b^2)r)}{4-(2-b)(2+b)^2r} \tag{36}$$

$$\pi_i^C = \frac{(a-c)^2r((4-b^2)^2r-8)}{(4-(2-b)(2+b)^2r)^2} \tag{37}$$

$$W^C = \frac{(a-c)^2r((3+b)(4-b^2)^2r-16)}{(4-(2-b)(2+b)^2r)^2} \tag{38}$$

Then, it is easy to obtain the comparisons from Appendix B, i.e., $x_i^{CP} > x_i^F > x_i^C$, $q_i^F > q_i^{CP} > q_i^C$, and $W^F > W^{CP} > W^C$ for $b \in (0,1)$.

Proof of lemma 2.

If we set $s^P = 0$ in (18) and (19), then we have the following equilibrium outcomes:

$$x_i^B = \frac{2(2-b^2)(a-c)}{b^2(2-6r)+8r+4br-b^3r+b^4r-4} \tag{39}$$

$$q_i^B = \frac{(4-b^2)(a-c)r}{b^2(2-6r)+8r+4br-b^3r+b^4r-4} \tag{40}$$

$$p_i^B = \frac{(4+4b-b^2-b^3)cr+a(b^2(2-5r)-4(1-r)+b^4r)}{b^2(2-6r)+8r+4br-b^3r+b^4r-4} \tag{41}$$

$$\pi_i^B = \frac{(a-c)^2r(8+b^4(2-9r)-16r+b^6r-8b^2(1-3r))}{(4-b^2(2-6r)-8r-4br+b^3r-b^4r)^2} \tag{42}$$

$$W^B = \frac{(a-c)^2r(16+b^4(4-19r)-48r-16br+8b^3r-b^5r+2b^6r-8b^2(2-7r))}{(4-b^2(2-6r)-8r-4br+b^3r-b^4r)^2} \tag{43}$$

Then, it is easy to obtain the comparisons from Appendix B, i.e., $x_i^F > x_i^{BP} > x_i^B$, $q_i^F > q_i^{BP} > q_i^B$, and $W^F > W^{BP} > W^B$ for $b \in (0,1)$.

Proof of proposition 1.

$$s^{CP} - s^{BP} = \frac{b^2(-4+b^2)(a-c)(2+(-4+b^2)r)}{16(-1+r)-8b^2(-1+r)+16br-8b^3r+b^4r+b^5r} > 0$$

Proof of proposition 2.

$$x_i^{CP} - x_i^{BP} = \frac{2b^2(4-b^2)(a-c)}{16(-1+r)-8b^2(-1+r)+16br-8b^3r+b^4r+b^5r} > 0$$

Proof of proposition 3.

$$q_i^{CP} - q_i^{BP} = 0,\ p_i^{CP} - p_i^{BP} = 0$$

Proof of proposition 4.

$$\pi_i^{CP} - \pi_i^{BP} = \frac{b^2(-4+b^2)^3(a-c)^2r(2+(-4+b^2)r)}{(16(-1+r)-8b^2(-1+r)+16br-8b^3r+b^4r+b^5r)^2} > 0$$

Proof of proposition 5.

$$W^{CP} - W^{BP} = 0$$

Second, with R&D subsidies, by comparing R9 to R16, we obtain the ranges of *r* and *b* when comparing equilibrium outcomes: $r > \frac{a(3-2b)}{(2-b)^2(1+b)c}$ if $b \le b_2$, $r > \frac{2(2-b^2)}{(2+b)^2(2-3b+b^2)}$ if $b > b_2$, where $b_2 =$

$$\frac{9a+6c+\sqrt{3}(-a+c)V+\sqrt{6}(a-c)\sqrt{\frac{16(9a-8c)}{a-c}+\frac{3(3a+2c)^2}{(a-c)^2}+\frac{3\sqrt{3}(13a^3-174a^2c+44ac^2-8c^3)}{(a-c)^3V}-\frac{4((a-c)^3)^{1/3}(441a^2-540ac+280c^2)}{(a-c)^2Z^{1/3}}-\frac{4Z^{1/3}}{((a-c)^3)^{1/3}}}}{-24(a-c)}$$

$V = \sqrt{\frac{16(9a-8c)}{a-c}+\frac{3(3a+2c)^2}{(a-c)^2}+\frac{8((a-c)^3)^{1/3}(441a^2-540ac+280c^2)}{(a-c)^2Z^{1/3}}+\frac{8Z^{1/3}}{((a-c)^3)^{1/3}}}$ and $Z = (9261a^3 - 17226a^2c + 13716ac^2 - 4672c^3 + 6\sqrt{3}\sqrt{-c(37044a^5 - 15111a^4c + 14252a^3c^2 - 3900a^2c^3 + 10688ac^4 + 1152c^5)})$

Then we have following proofs.

Proof of lemma 3.

From Eqs. (34) to (38), then, it is easy to obtain the comparisons from Appendix B, i.e., $x_i^F > x_i^{CR} > x_i^C$, $q_i^F > q_i^{CR} > q_i^C$, and $W^F > W^{CR} > W^C$ for $b \in (0,1)$.

Proof of lemma 4.

From Eqs. (39) to (43), then, it is easy to obtain the comparisons from Appendix B, i.e., $x_i^F > x_i^{BR} > x_i^B$, $q_i^F > q_i^{BR} > q_i^B$.and $W^F > W^{BR} > W^B$ for $b \in (0,1)$.

Proof of proposition 6.

$$s^{CR} - s^{BR} = \frac{b^2(a-c)r\left(-8+2b+b^2(2-8r)+16r+b^4r\right)}{(-2+b)(2+b)(-3+2b+4r-3b^2r+b^3r)\left(-3+4r+b^2r+b(-1+4r)\right)} < 0$$

Proof of proposition 7.

$$x_i^{CR} - x_i^{BR} = \frac{b^2(-4+2b+b^2)(a-c)r}{(-3+2b+4r-3b^2r+b^3r)\left(-3+4r+b^2r+b(-1+4r)\right)} < 0$$

Proof of proposition 8.

$$q_i^{CR} - q_i^{BR} = \frac{b^2(a-c)r(1+(-4+b^2)r)}{(-3+2b+4r-3b^2r+b^3r)\left(-3+4r+b^2r+b(-1+4r)\right)} < 0$$

$$p_i^{CR} - p_i^{BR} = -\frac{b^2(1+b)(a-c)r(1+(-4+b^2)r)}{(-3+2b+4r-3b^2r+b^3r)\left(-3+4r+b^2r+b(-1+4r)\right)} > 0$$

Proof of proposition 9.

$$\pi_i^{CR} - \pi_i^{BR} = \frac{\left\{\begin{array}{c} b^2(a-c)^2r(4b^7(-1+r)r^2+b^8r^2(1+4r)+2b^5r(1+32r-24r^2) \\ +48(3-10r+8r^2)-2b^6r(-3+7r+24r^2)-8b^2(7-46r+36r^2+32r^3) \\ +4b^3(5-4r-72r^2+48r^3)-4b(21-8r-96r^2+64r^3)+2b^4(4-43r+44r^2+96r^3)) \end{array}\right\}}{2(-4+b^2)(-3+2b+4r-3b^2r+b^3r)^2(-3+4r+b^2r+b(-1+4r))^2} \gtrless 0$$

Proof of proposition 10.

$$W^{CR} - W^{BR} = \frac{b^2(-4+2b+b^2)(a-c)^2r^2}{(-3+2b+4r-3b^2r+b^3r)\left(-3+4r+b^2r+b(-1+4r)\right)} < 0$$

Finally, we compare the output and R&D subsidies under Cournot or Bertrand competitions, respectively. Then, by comparing R1 to R16, we obtain the following ranges:

(1) The ranges of *r* and *b* under Cournot competition when comparing output and R&D subsidies:

$$r > \frac{4}{(2-b)^2(2+b)} \text{ if } b \geq \frac{-3c+\sqrt{32a^2+c^2}}{2(2a+c)};\ r > \frac{4\left(a(4-b^2)-b^2c\right)}{(1+b)(4-b^2)^2c} \text{ if } b < \frac{-3c+\sqrt{32a^2+c^2}}{2(2a+c)}.$$

(2) The ranges of *r* and *b* under Bertrand competition when comparing output and R&D subsidies:

$$r > \frac{2(2-b^2)}{(2+b)^2(2-3b+b^2)} \text{ if } b \geq b_3;\ r > \frac{8a(2-b^2)}{(1+b)(4-b^2)^2c} \text{ if } b < b_3.$$

Then we have the following proofs:

Proof of proposition 11

(i) $$s^{CP} - s^{CR} = \frac{(a-c)(4b^2(-6+b+b^2)+64r-b^3(-16+b(2+b(5+b)))r-3(-2+b)^2(2+b)^3r^2)}{(-2+b)(-3-b+(2+b)^2r)(8(-2+b^2)+(1+b)(-4+b^2)^2r)} > 0$$

$$s^{BP} - s^{BR} = \frac{(a-c)(b^2(192-b^2(142+b(3-b(40+b-4b^2))))r-2b^2(2+b)(3-2b)(2-b^2)-64r+(2-b)^3(1+b)(2+b)^2(3-5b^2+b^4)r^2)}{(2+b)(-3+2b+(-2+b)^2(1+b)r)(8(-2+b^2)+(1+b)(-4+b^2)^2r)} \gtrless 0 \text{ if } b \gtrless b_3.$$

(ii) $$x_i^{CP} - x_i^{CR} = \frac{(a-c)(-4b^2(3+b)-(2+b)^2(-4+b(4-5b+b^3))r)}{(-3-b+(2+b)^2r)(8(-2+b^2)+(1+b)(-4+b^2)^2r)} > 0$$

$$x_i^{BP} - x_i^{BR} = \frac{(a-c)(2b^2(-3+2b)(-2+b^2)+(-2+b)^2(1+b)(2+b)(2+b(-3+2(-1+b)b))r)}{(-3+2b+(-2+b)^2(1+b)r)(8(-2+b^2)+(1+b)(-4+b^2)^2 r)} \gtrless 0 \text{ if } b \gtrless b_3.$$

(iii) $$q_i^{CP} - q_i^{CR} = \frac{(a-c)r(-16+8b^2-3b^4-b^5+(-2+b)^2(2+b)^3 r)}{(-3-b+(2+b)^2 r)(8(-2+b^2)+(1+b)(-4+b^2)^2 r)} > 0$$

$$q_i^{BP} - q_i^{BR} = \frac{(-2+b)(a-c)r(8-4b-6b^2+b^3+2b^4+(-4+b^2)^2(-1+b^2)r)}{(-3+2b+(-2+b)^2(1+b)r)(8(-2+b^2)+(1+b)(-4+b^2)^2 r)} > 0.$$

Proof of propositions 12

It is difficult to find the range of $\hat{b}$ and $\hat{r}$, therefore, we provide a numerical simulation in a table and test the specific numbers as examples, which supports the possible outcomes, for easy understanding:

Table 1: W and π_i between output and R&D subsidies with Bertrand firms

Let $a = 10$ and $c = 7$	
b	W^{BP} vs. W^{BR}; π_i^{BP} vs. π_i^{BR}
0.01	$W^{BP} > W^{BR}$; $\pi_i^{BP} > \pi_i^{BR}$
0.1	$W^{BP} > W^{BR}$; $\pi_i^{BP} > \pi_i^{BR}$
0.2	$W^{BP} > W^{BR}$; $\pi_i^{BP} > \pi_i^{BR}$
0.3	$W^{BP} > W^{BR}$; $\pi_i^{BP} > \pi_i^{BR}$
0.4	$W^{BP} > W^{BR}$; $\pi_i^{BP} > \pi_i^{BR}$
0.5	$W^{BP} > W^{BR}$; $\pi_i^{BP} > \pi_i^{BR}$
0.6	$W^{BP} > W^{BR}$; $\pi_i^{BP} > \pi_i^{BR}$
0.7	$W^{BP} > W^{BR}$; $\pi_i^{BP} > \pi_i^{BR}$
0.8	$W^{BP} > W^{BR}$; $\pi_i^{BP} > \pi_i^{BR}$
0.9	$W^{BP} \lesseqgtr W^{BR}$ if $r \lesseqgtr 4.072$; $\pi_i^{BP} > \pi_i^{BR}$
0.99	$W^{BP} < W^{BR}$; $\pi_i^{BP} \lesseqgtr \pi_i^{BR}$ if $r \lesseqgtr 156.274$